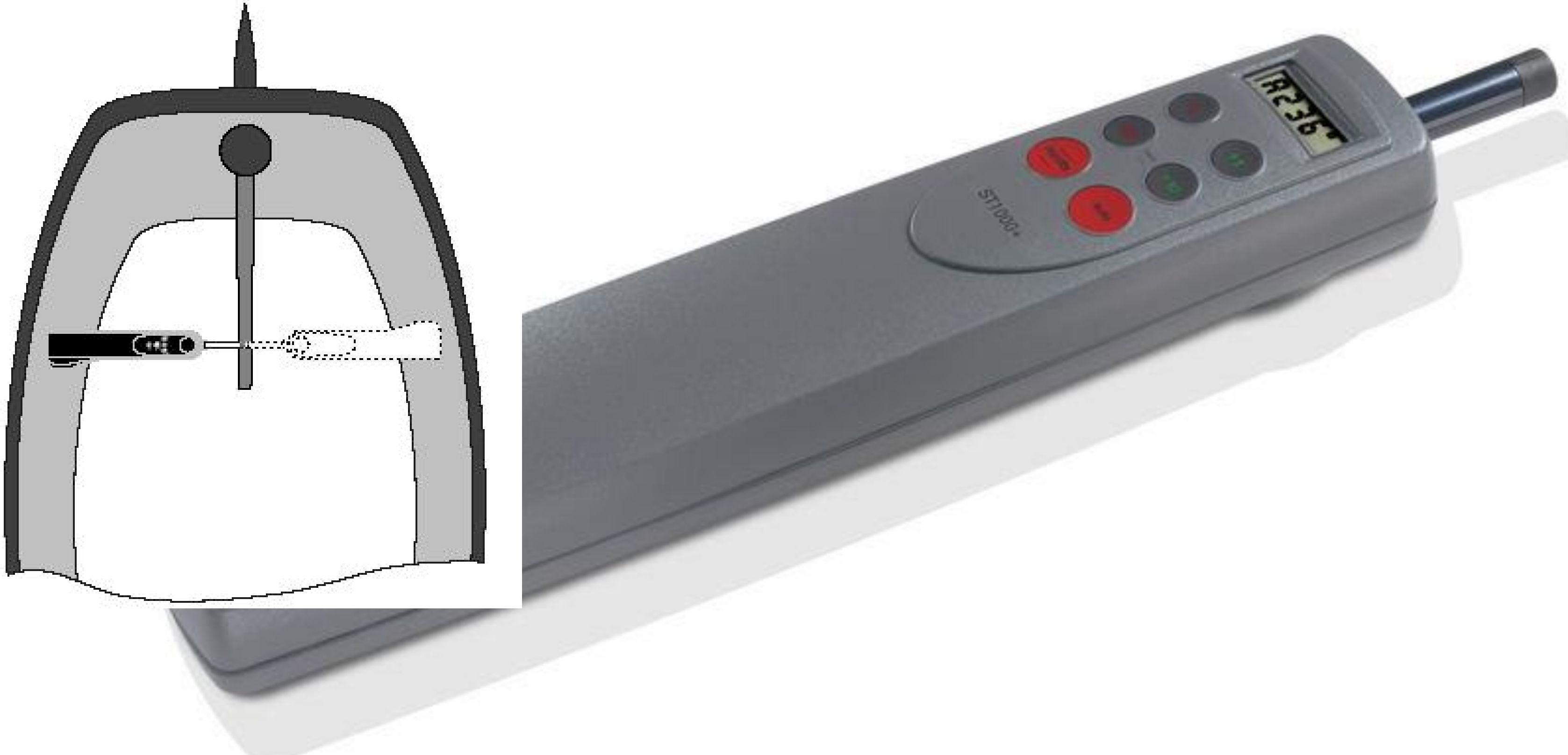
Introduction to Control Theory

Konstantin Tretyakov (kt@ut.ee)

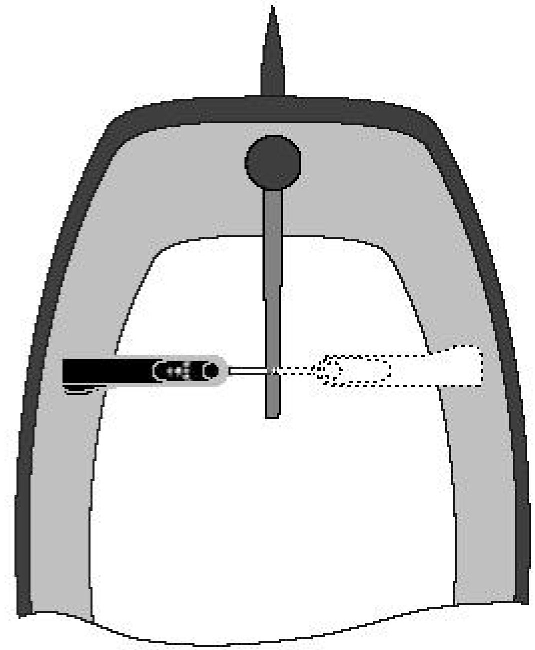
BIIT

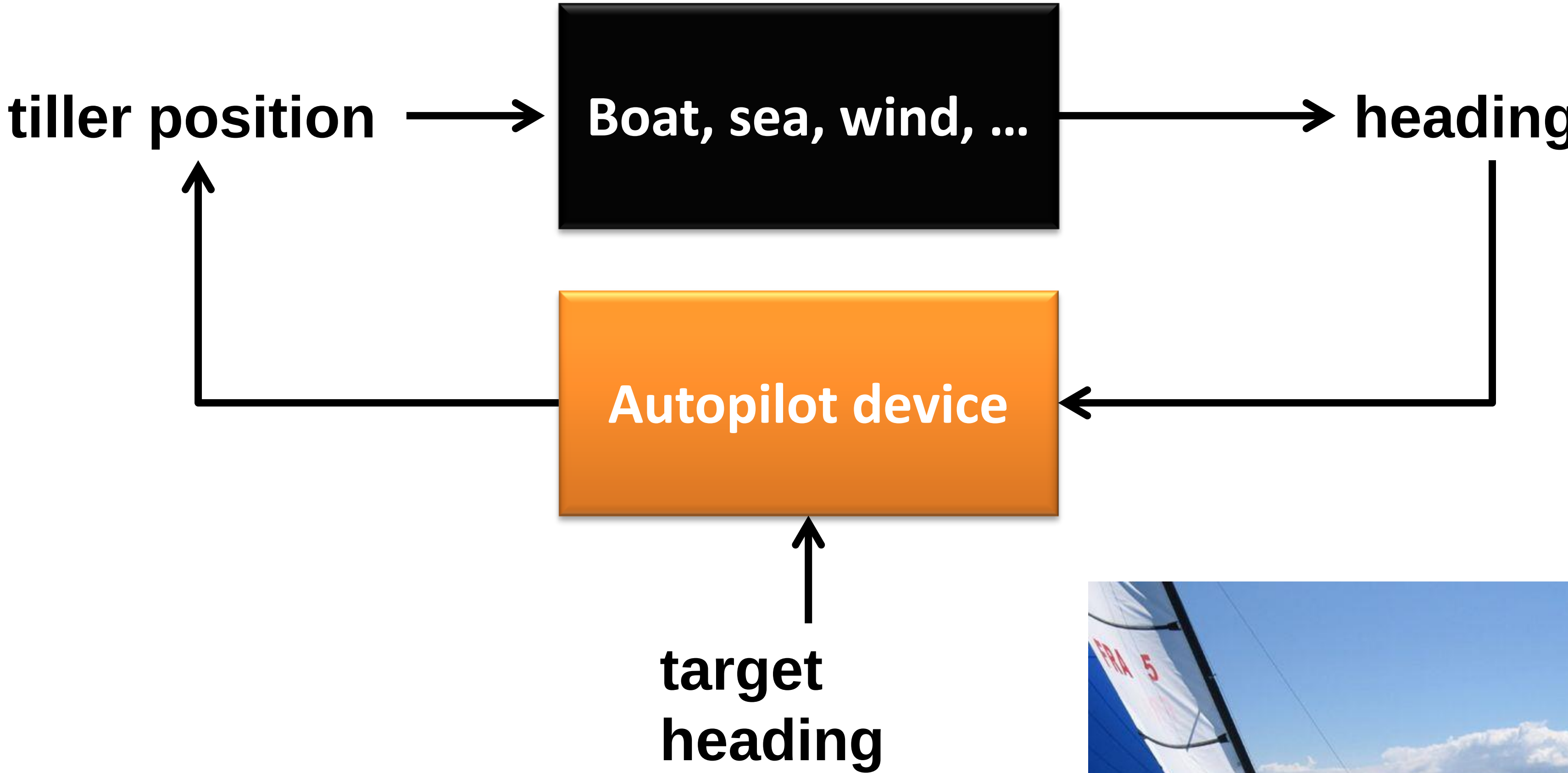
TARTU ÜLIKOOL · UNIVERSITAS TARTUENSIS 1632

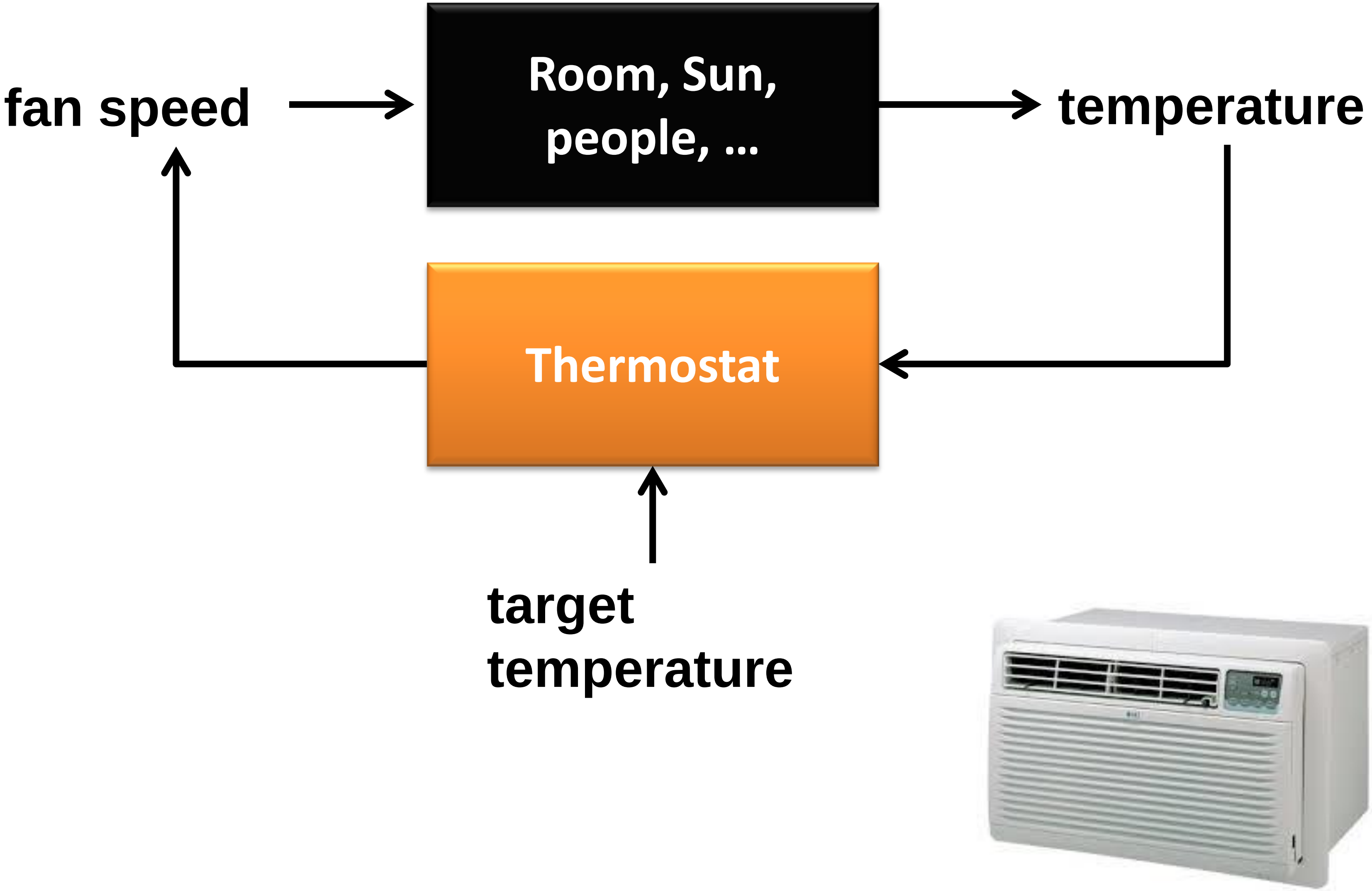

Raymarine
ST1000+

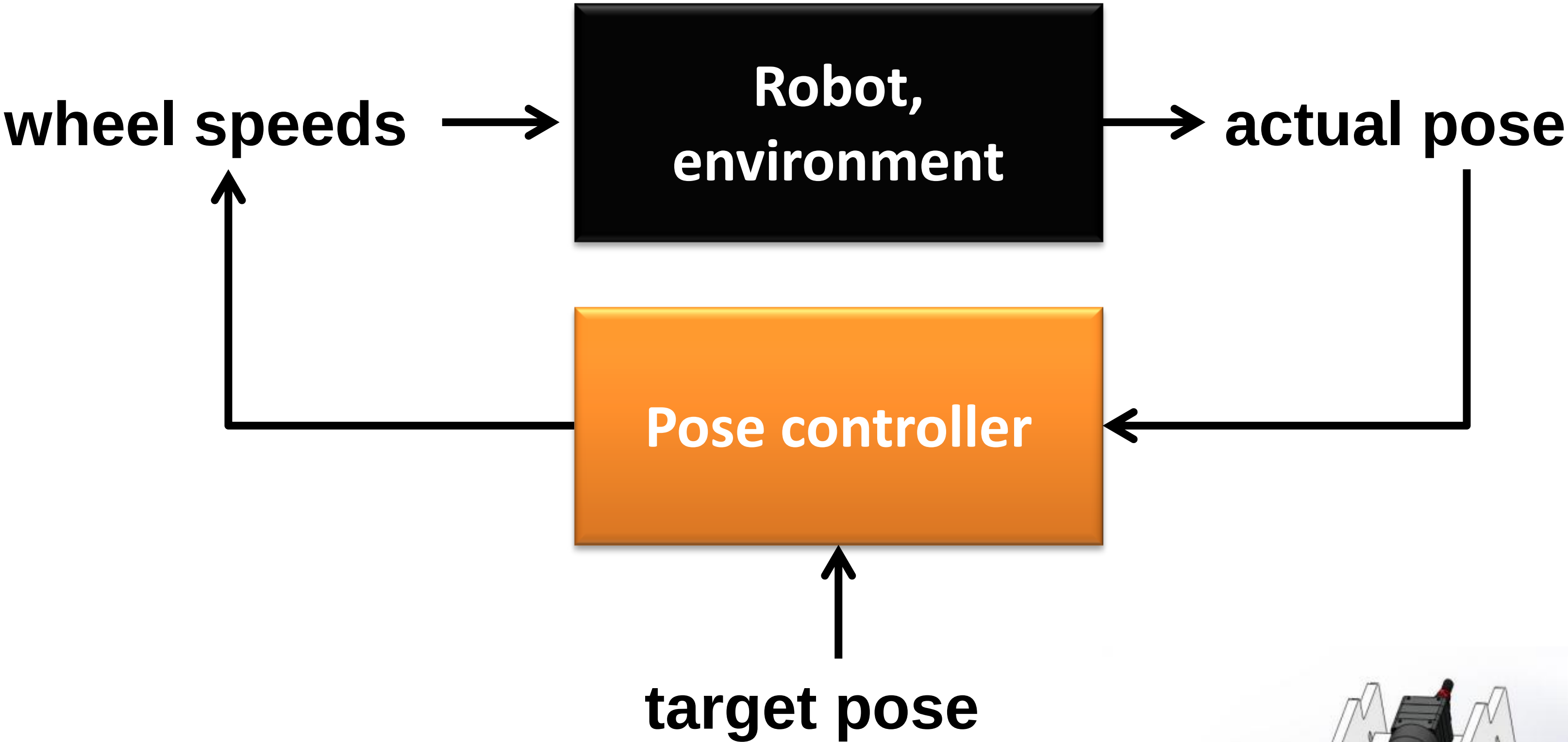


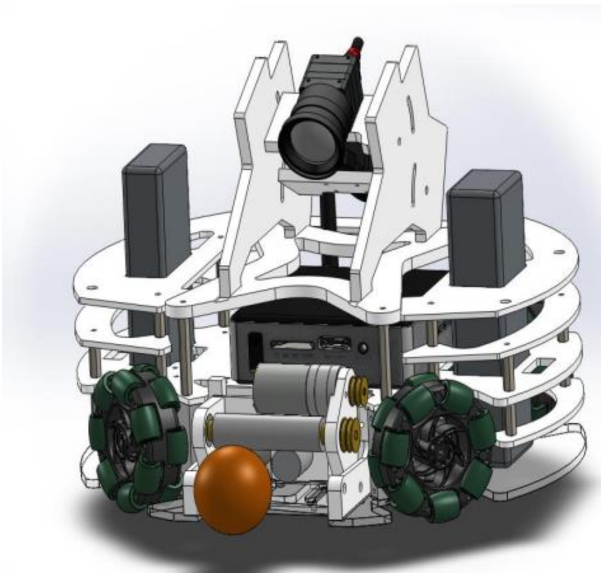
FRA 5

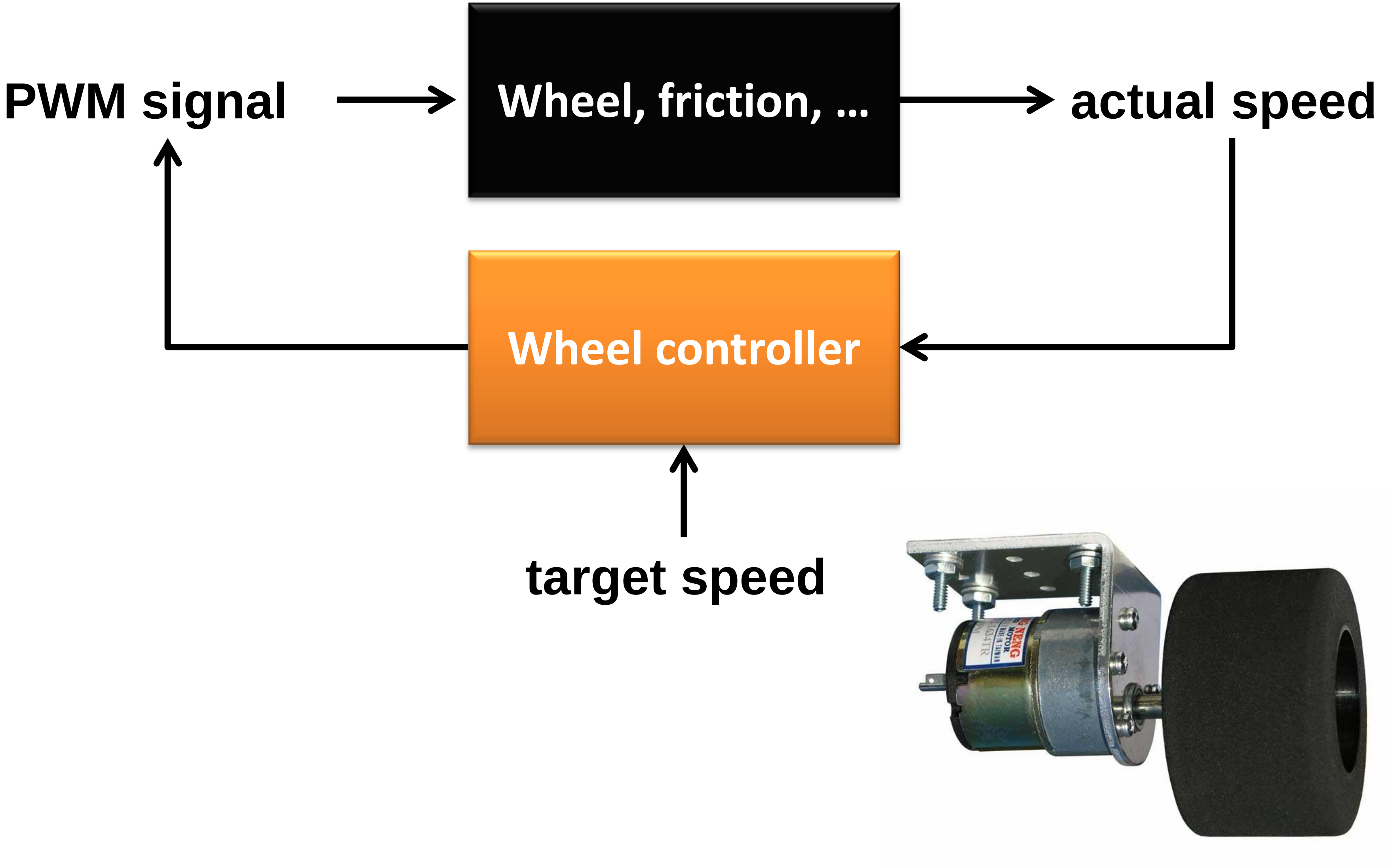


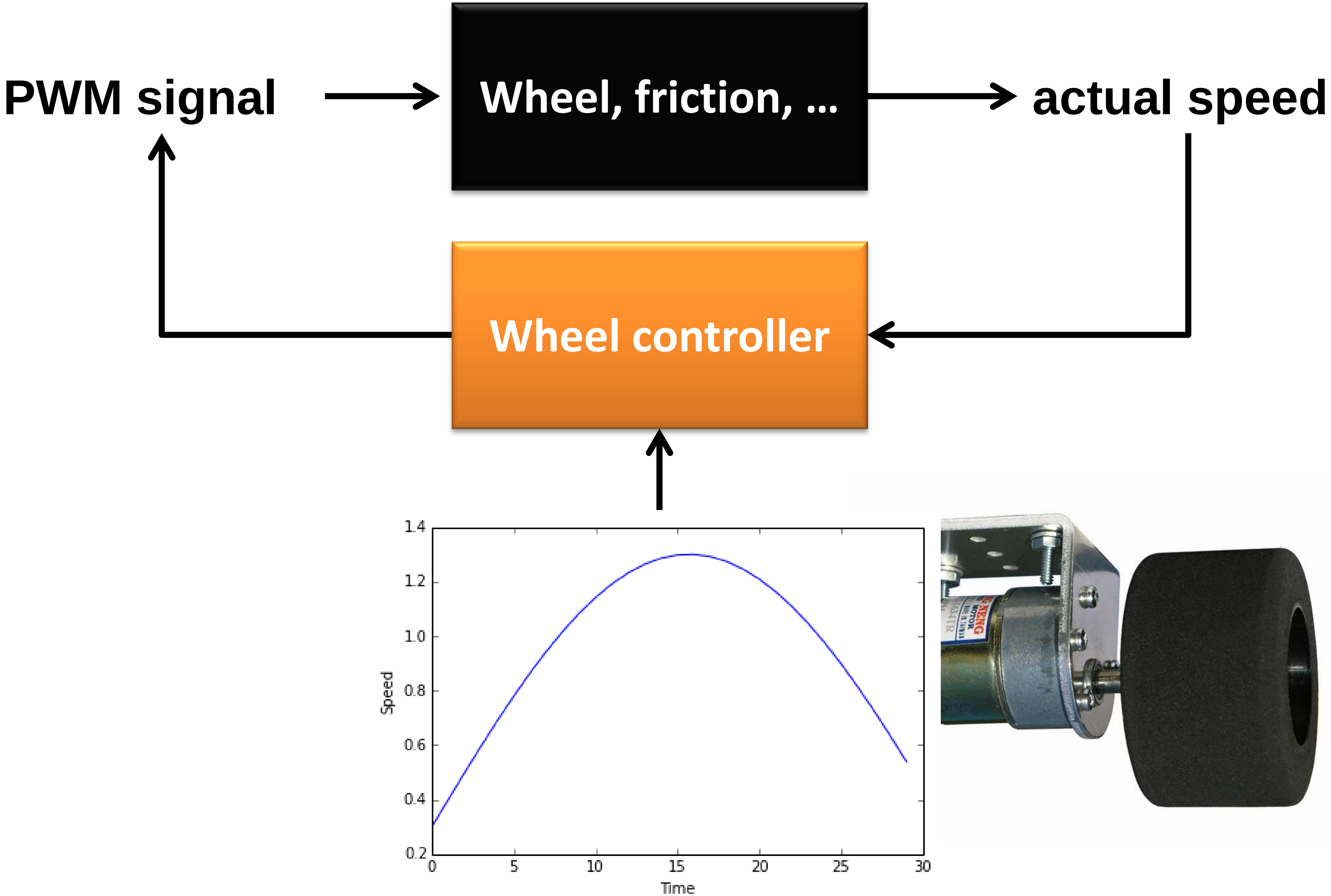

tiller position
Boat, sea, wind, ...
heading
Autopilot device
target heading

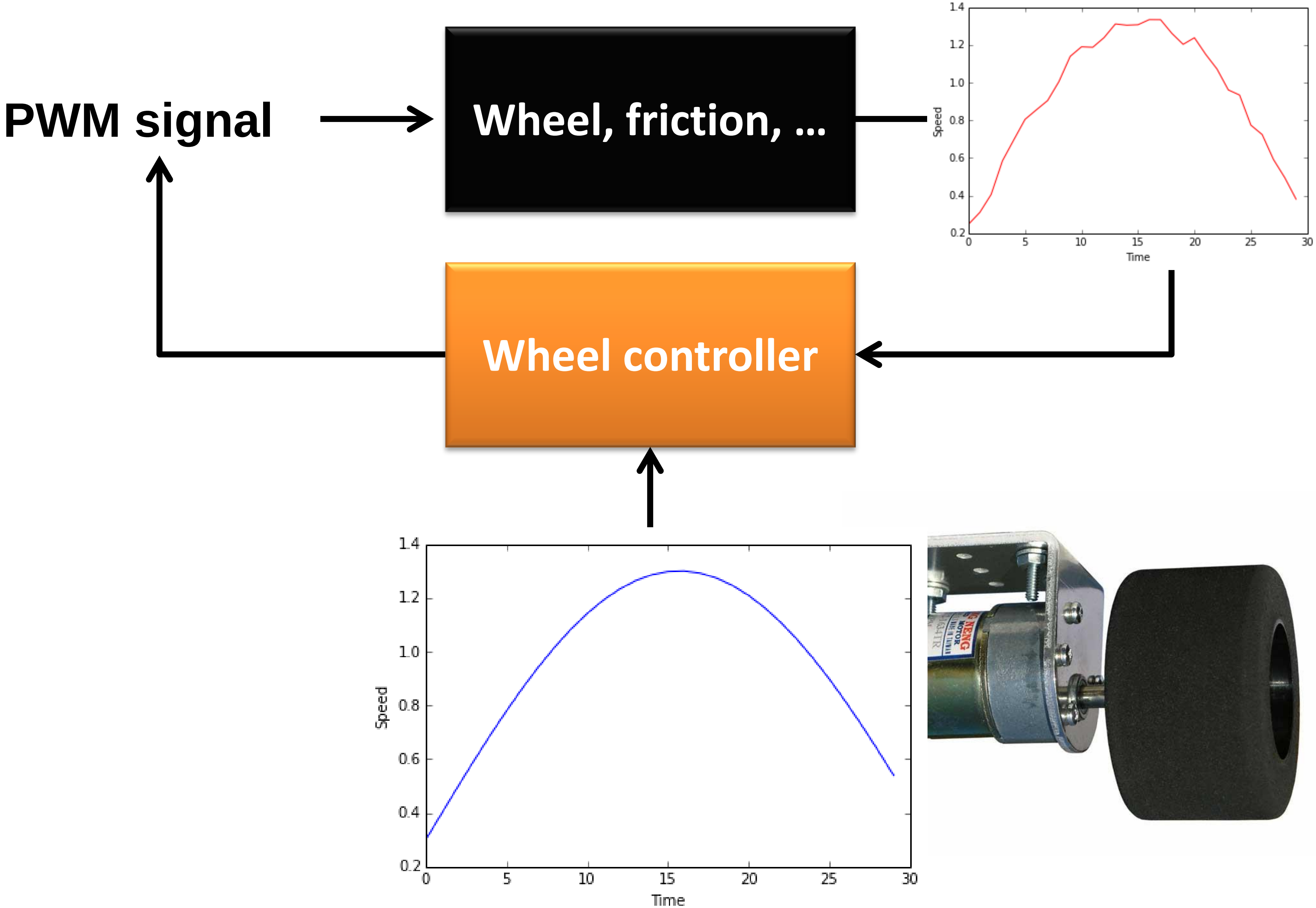

fan speed
Room, Sun, people, ...
temperature
Thermostat
target temperature

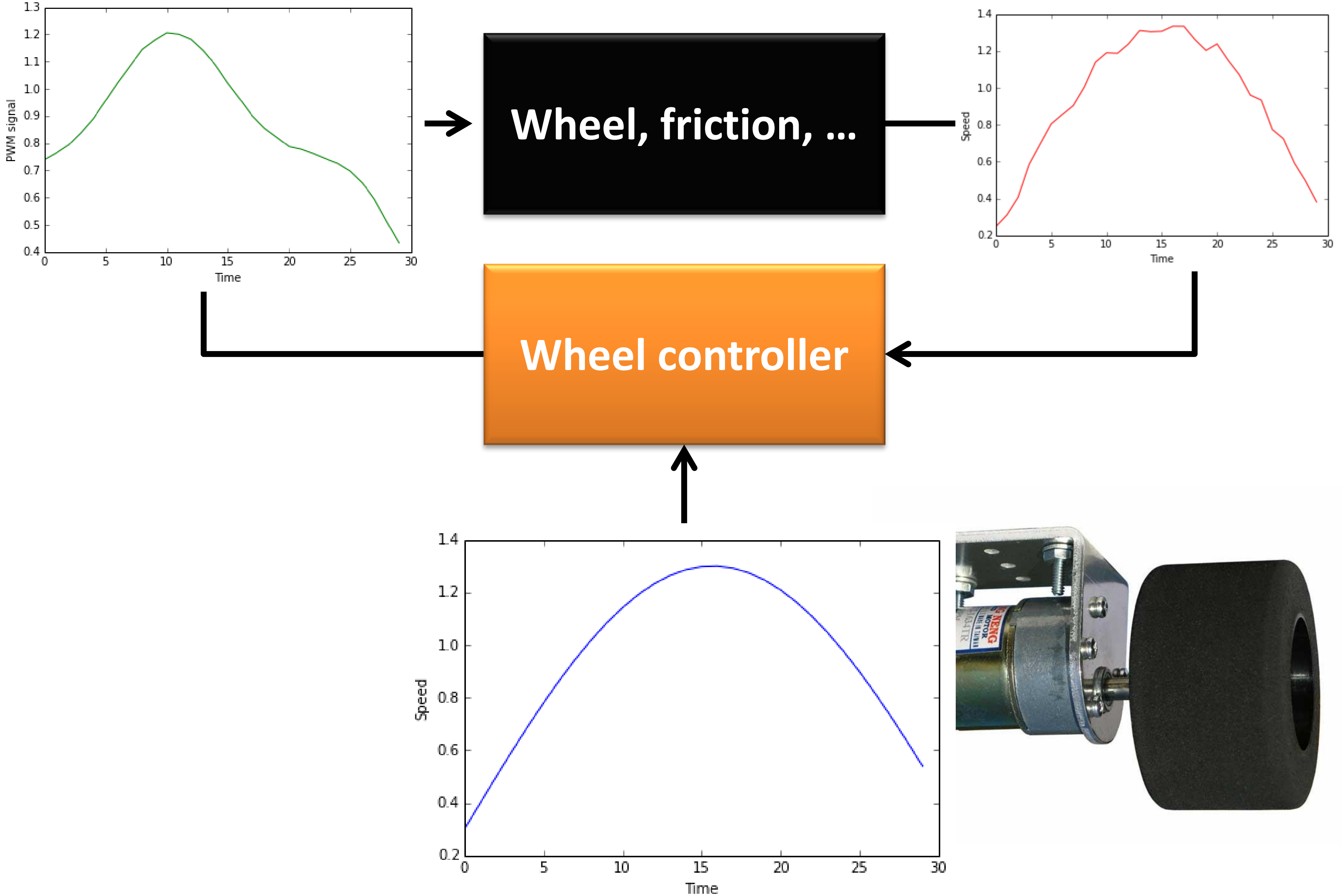

wheel speeds
Robot,
environment
actual pose
Pose controller
target pose

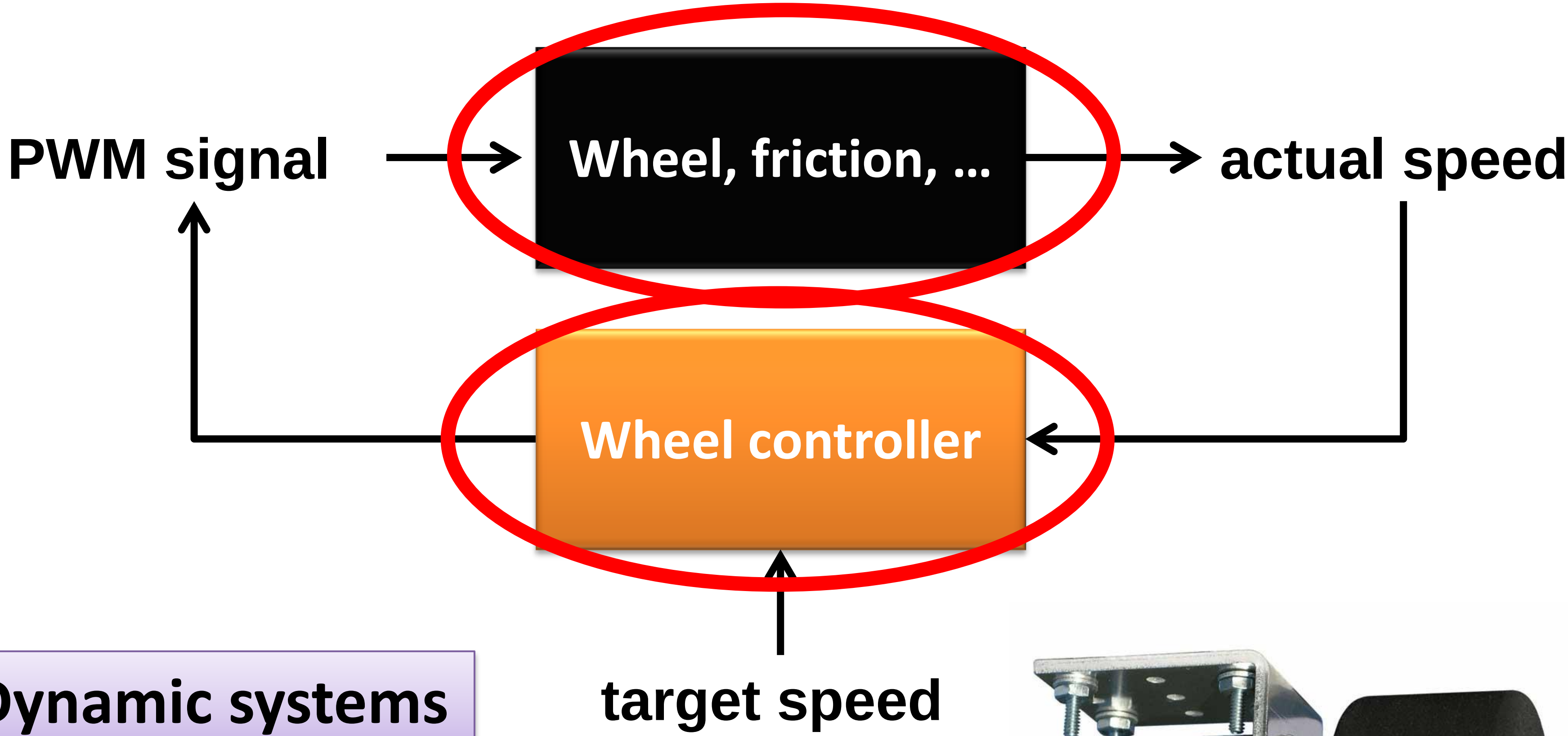



PWM signal
Wheel, friction, ...
actual speed
Wheel controller
target speed

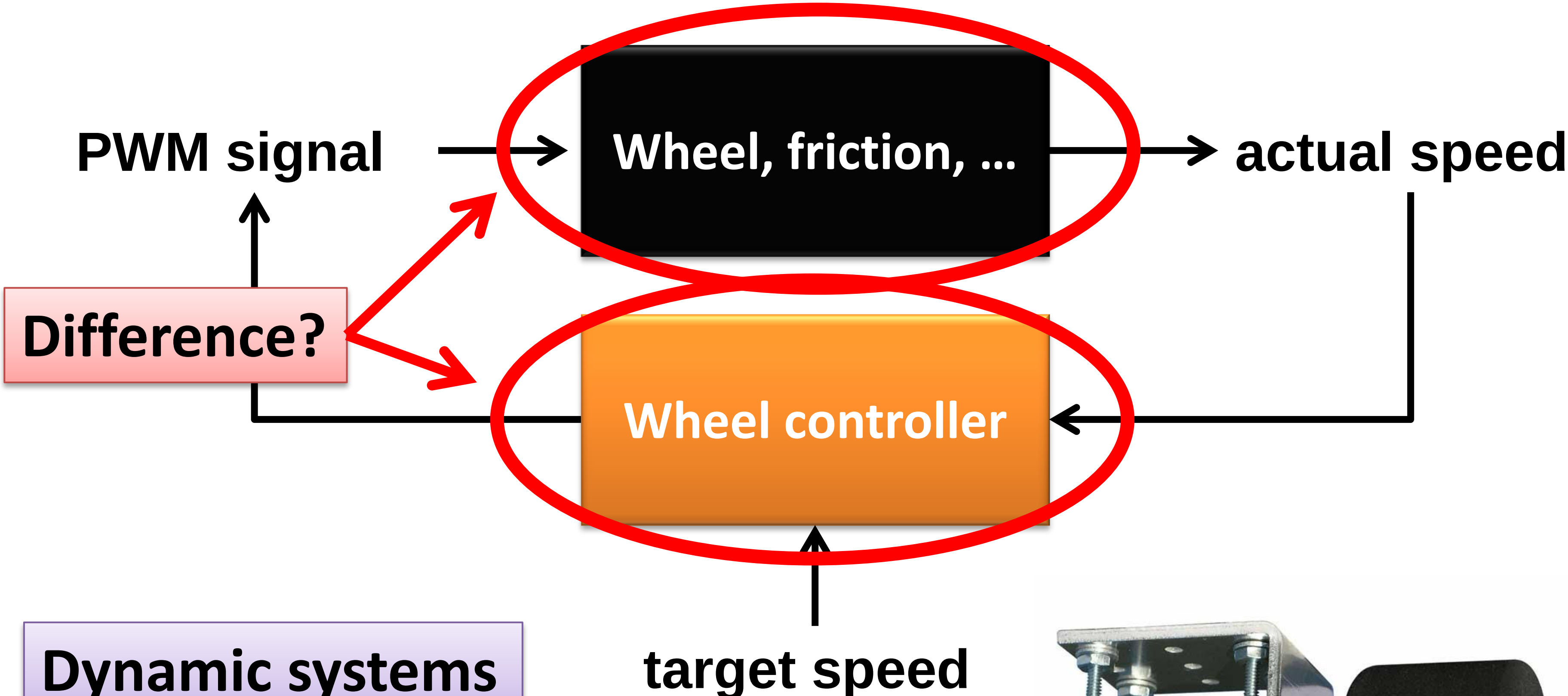

PWM signal
Wheel, friction, ...
actual speed
Wheel controller
Speed
Time
0.2
0.4
0.6
0.8
1.0
1.2
1.4
0
5
10
15
20
25
30


PWM signal
Wheel, friction, ...
Wheel controller
Speed
Time
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
5
10
15
20
25
30

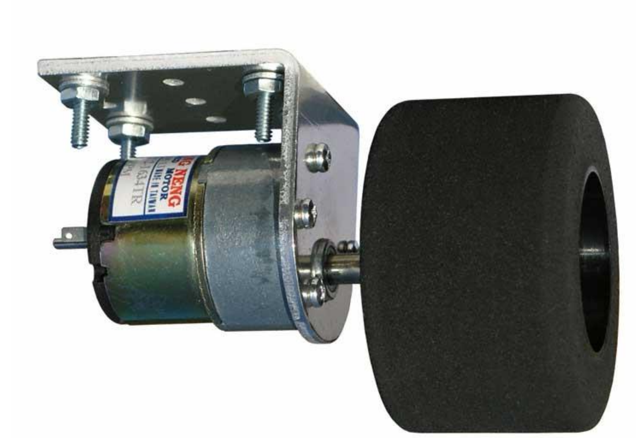

Wheel, friction, ...
Wheel controller
PWM signal
Time
Speed
Time
Speed
Time



Dynamic systems



Dynamic systems



System modeling & identification

PWM

ion, ...

actual speed

System control

eel controller

target speed

Dynamic systems

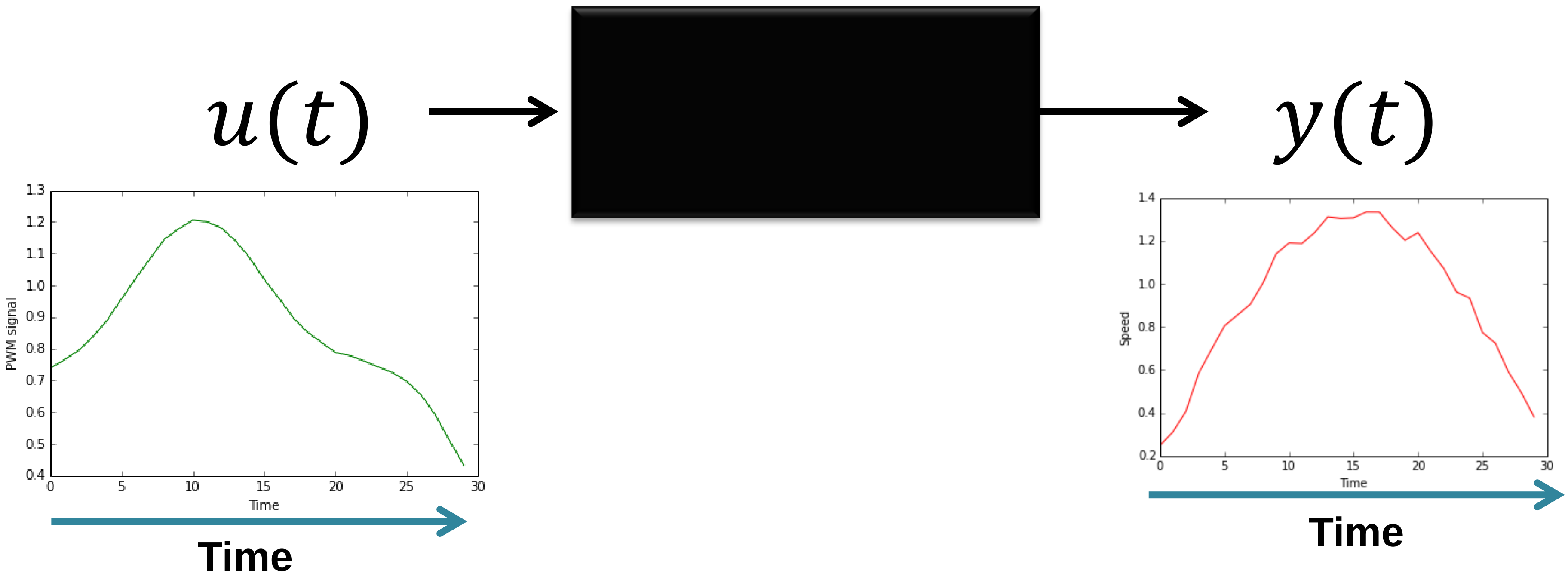
System modeling & identification

Dynamic system

Quiz

- How is a "**dynamic system**" different from a usual **function**?

Dynamic system

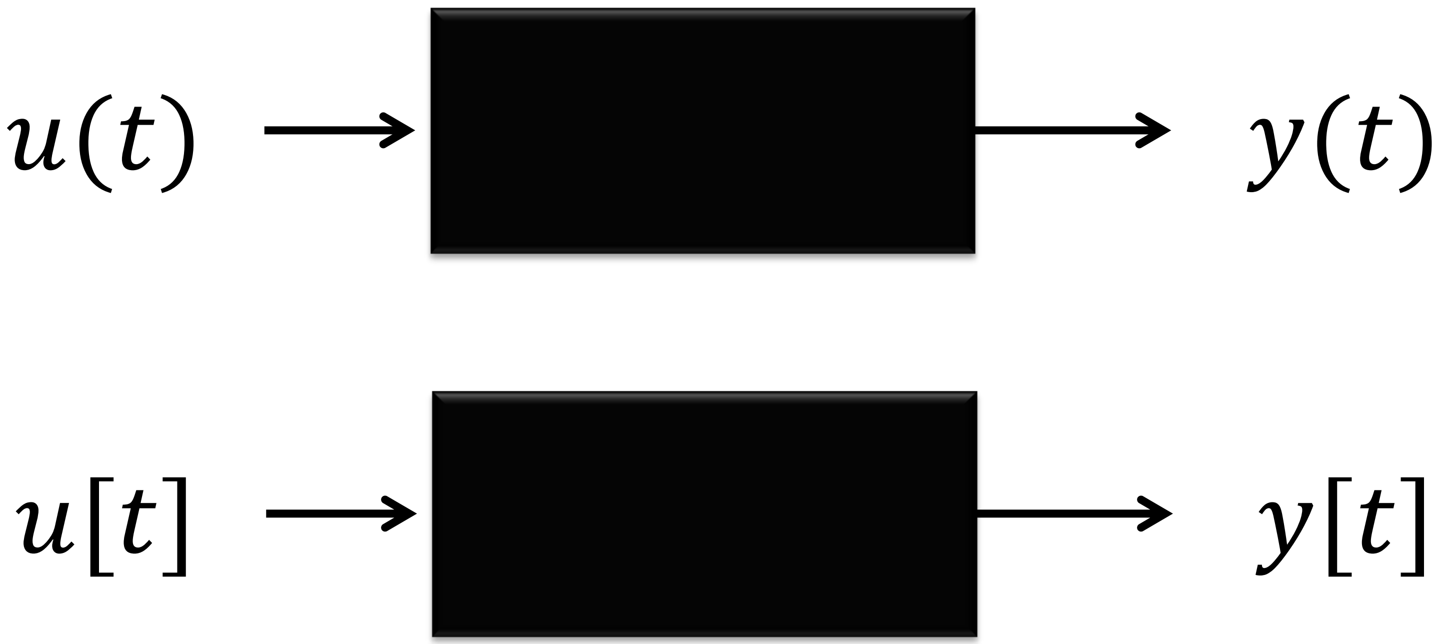


Terminology

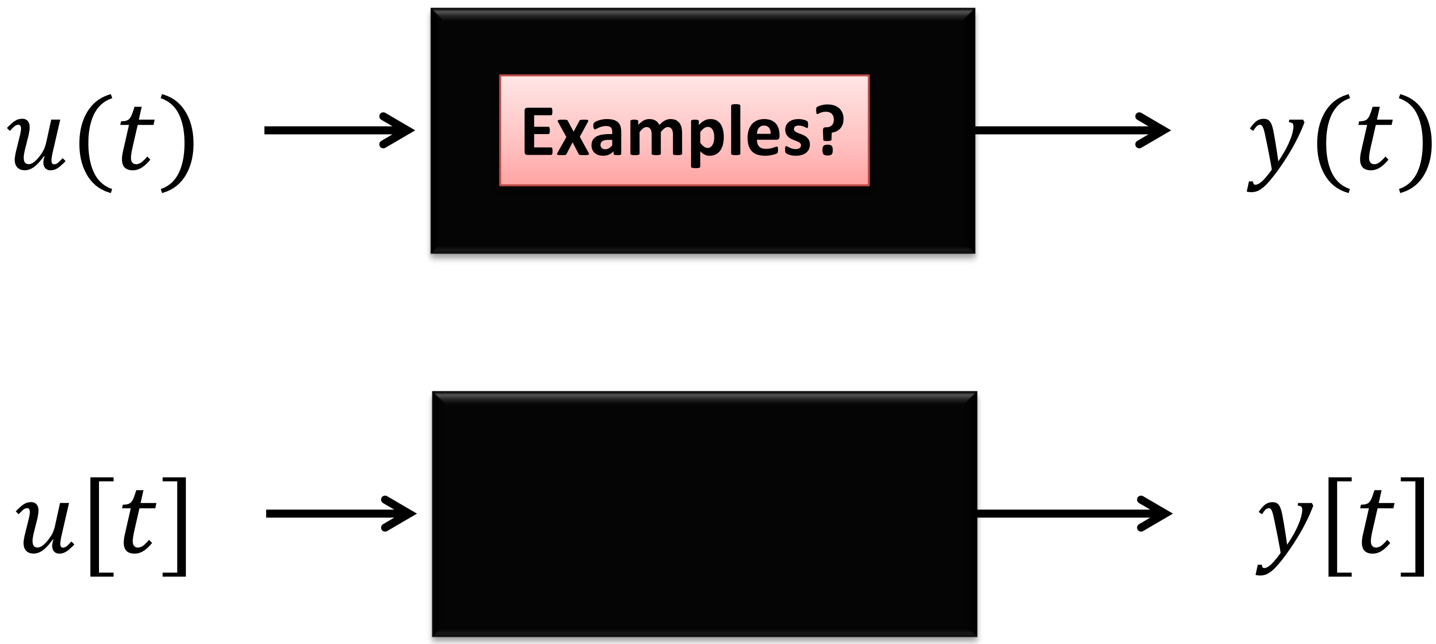
- continuous-time / discrete-time
- deterministic / probabilistic
- memoryless / with memory
- causal / noncausal
- time-invariant / time-varying
- finite-dimensional / infinite-dimensional
- linear / nonlinear

- stable / unstable
- controllable / noncontrollable
- observable / nonobservable

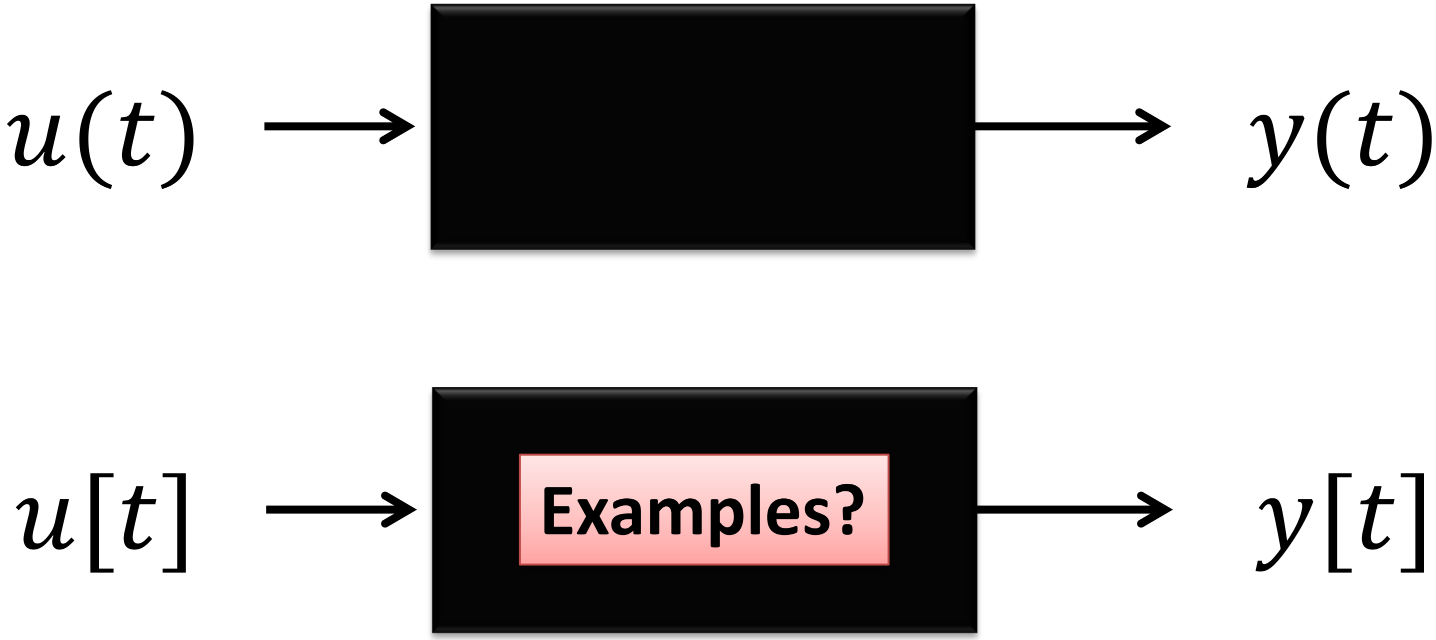
Continuous-time / Discrete-time



Continuous-time / Discrete-time

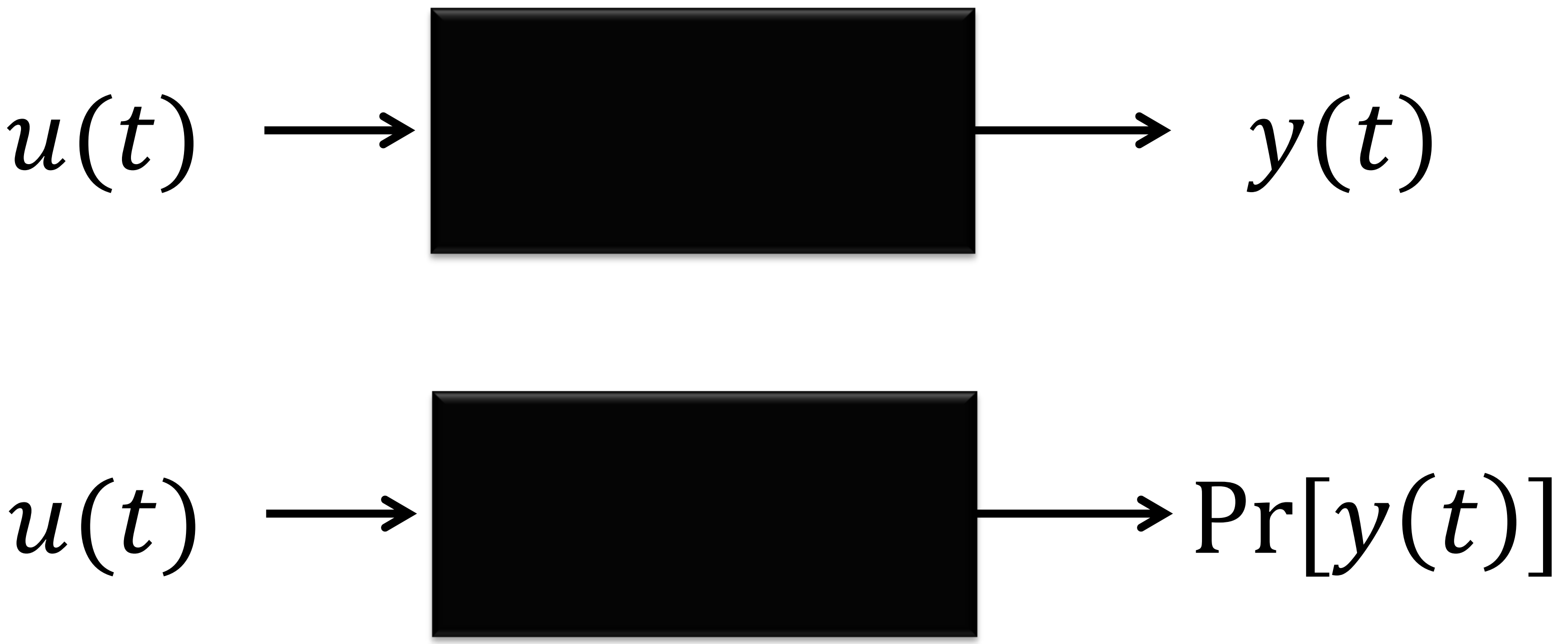


Continuous-time / Discrete-time

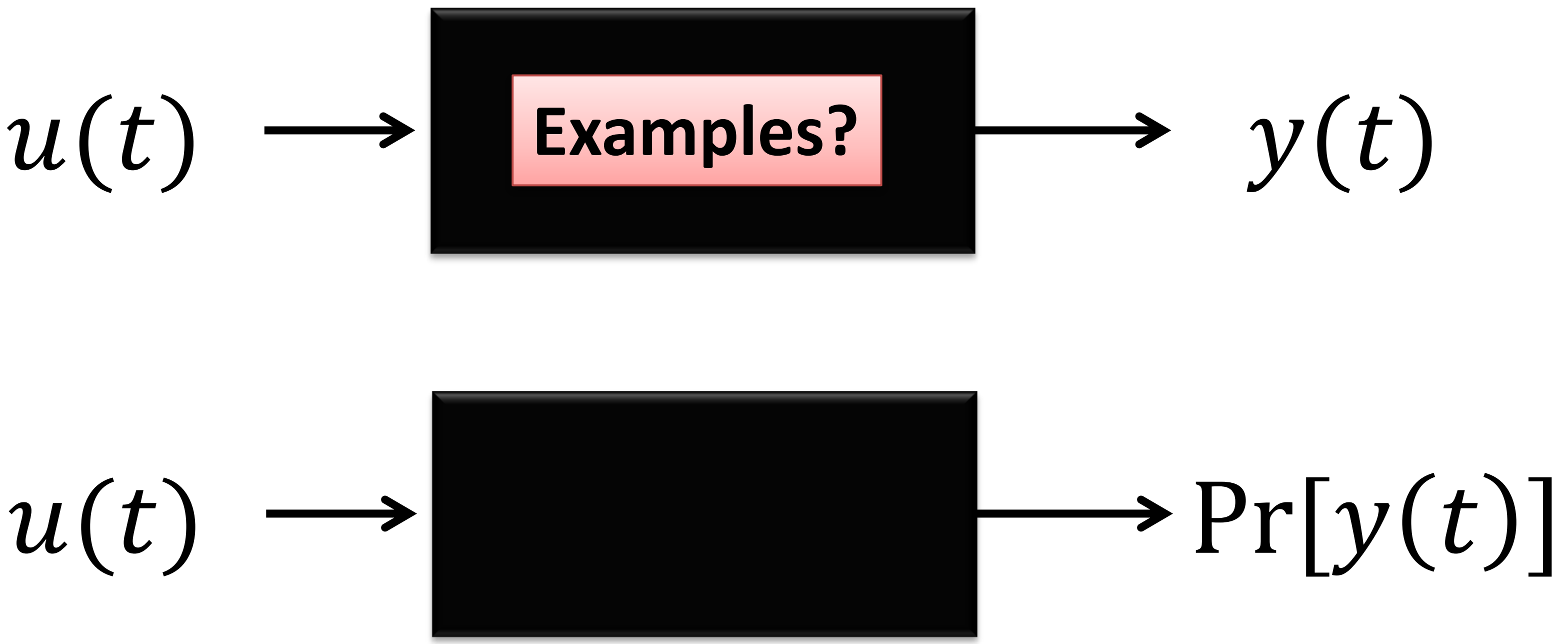


Discrete

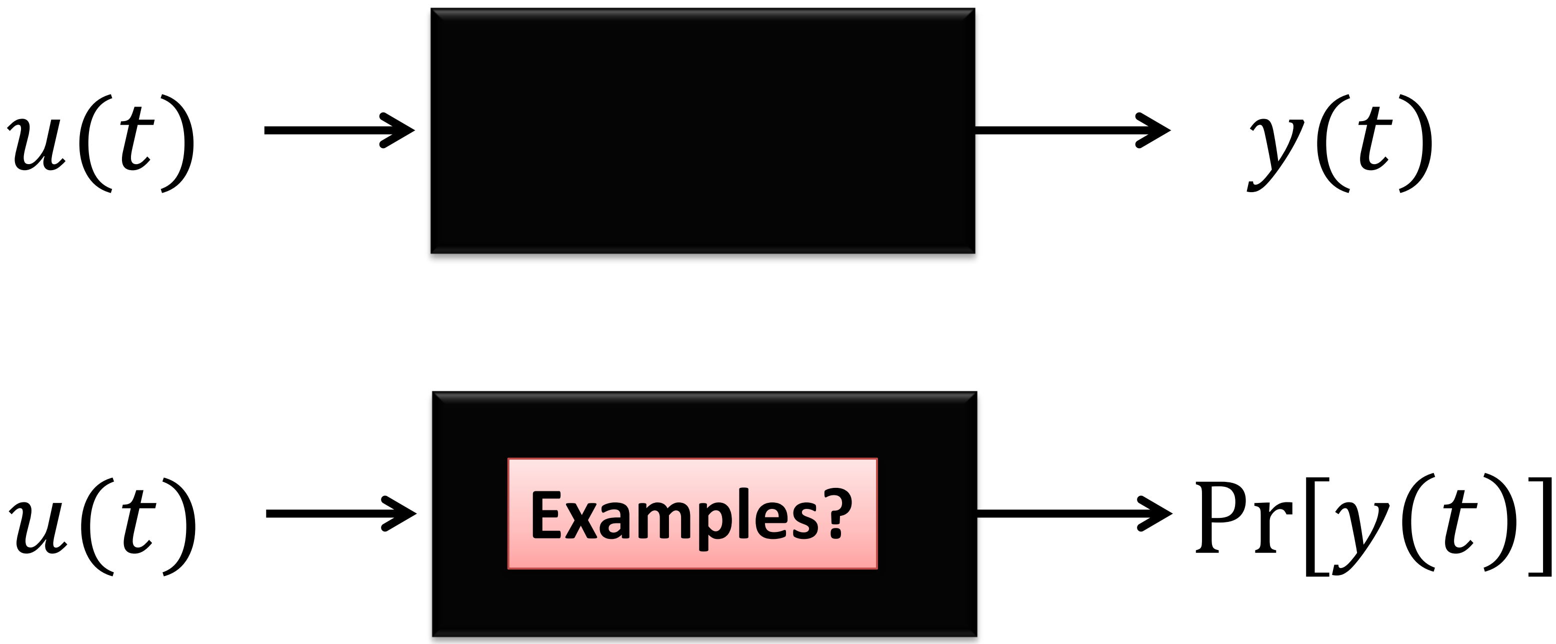
Deterministic / probabilistic



Deterministic / probabilistic

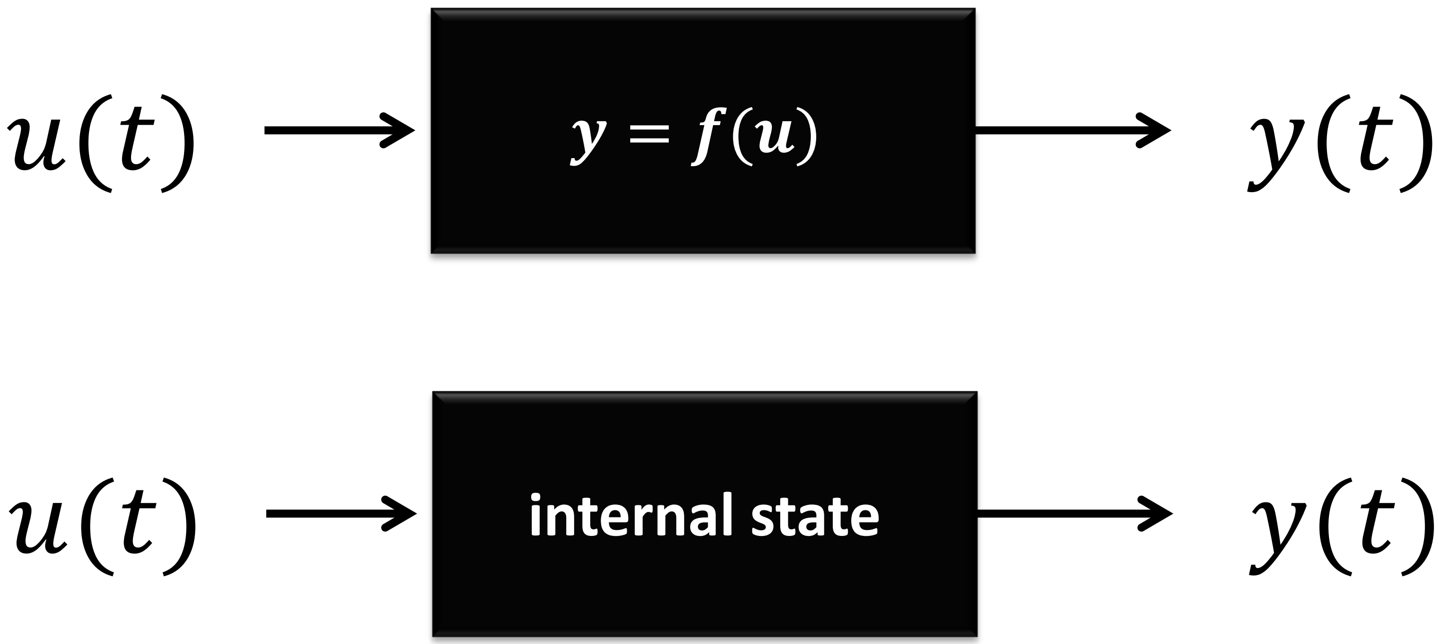


Deterministic / probabilistic

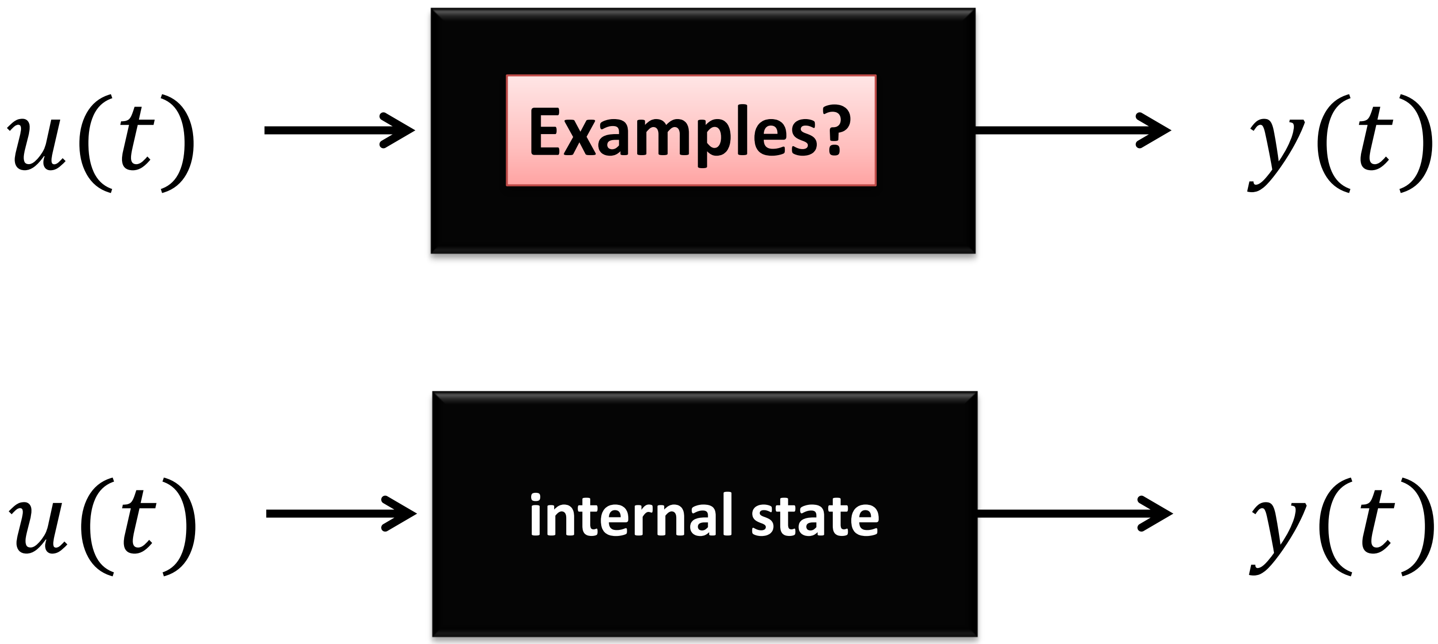


Discrete **Deterministic**

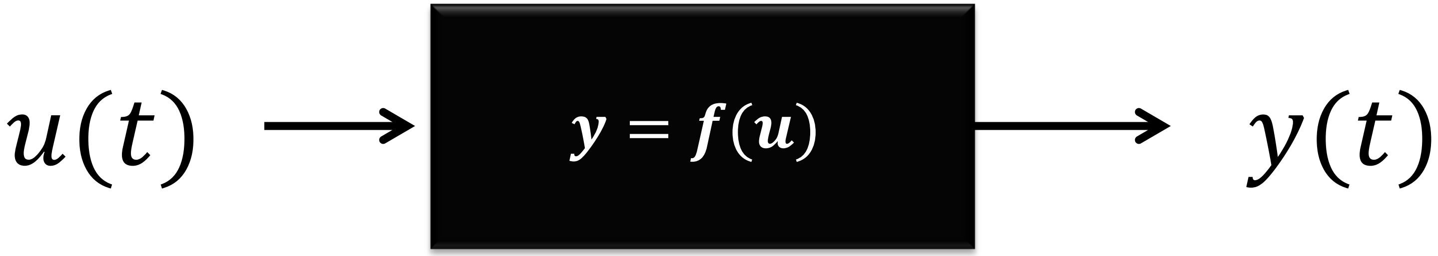
Memoryless / with memory

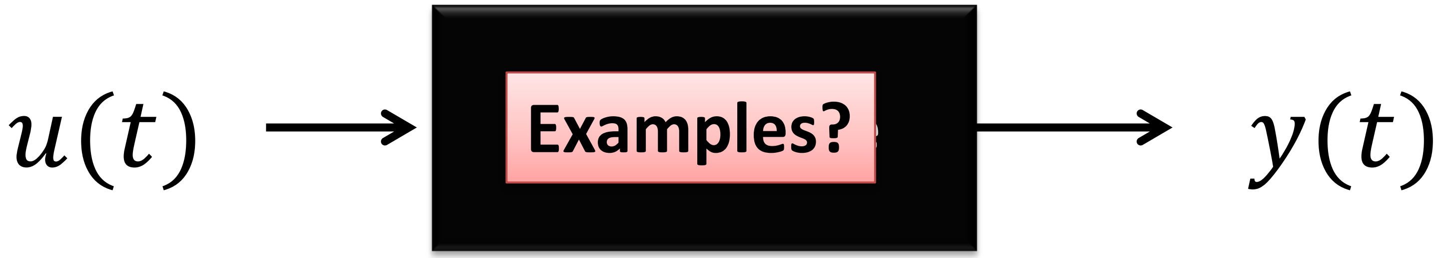


Memoryless / with memory



Memoryless / with memory





Discrete | **Deterministic** | **With memory**

Memory / state

In: [1, 2, 1, 0, 4, 5, 4, 5, 1, 2, 0, 1, 4, ...

Out: [a, a, b, b, a, c, d, d, e, a, f, ...

Time

Memory / state

In: [1, 2, 1, 0, 4, 5, 4, 5, 1, 2, 0, 1, 4, ...

Out: [a, a, b, b, a, c, d, d, e, a, f, ...

Time

Memory / state

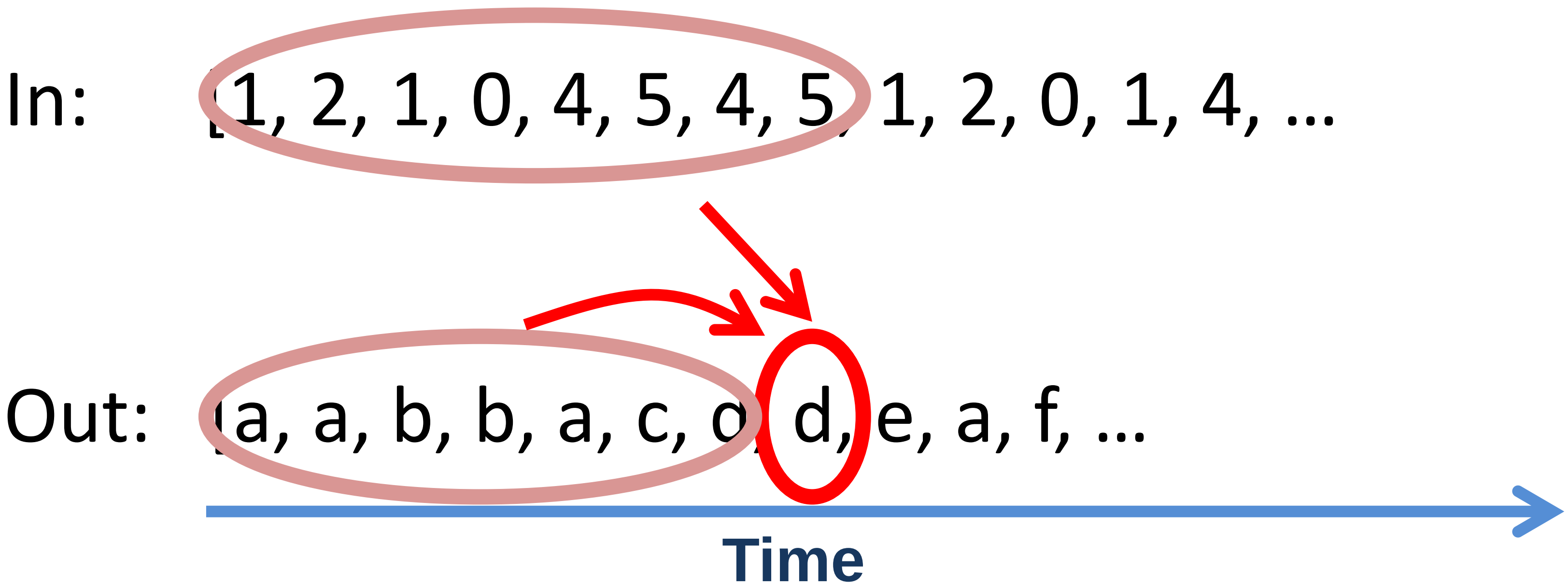
In: [1, 2, 1, 0, 4, 5, 4, 5, 1, 2, 0, 1, 4, ...

Out: [a, a, b, b, a, c, d, d, e, a, f, ...

Time

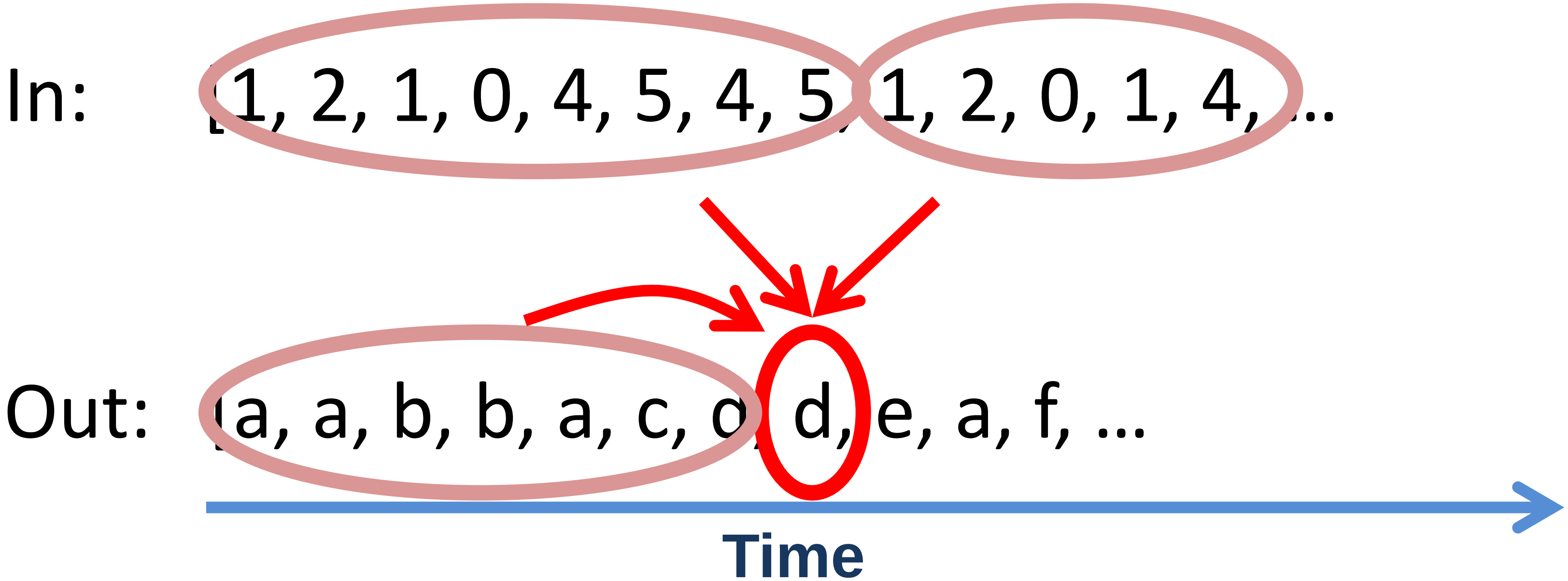
Memoryless (function)

Memory / state



Causal

Memory / state



Non-causal

Memory / state

In: [1, 2, 1, 0, 4, 5, 4, 5, 1, 2, 0, 1, 4, ...

Out: [a, a, b, b, a, c, d, d, e, a, f, ...

Time

Non-causal

Examples?

Memory / state

In: [1, 2, 1, 0, 4, 5, 4, 5, 1, 2, 0, 1, 4, ...

Out: [1, 1, 1, 1, 3, 4, 5, 3, 3, 1, 1, 2, 1, ...

Time

Moving average

Memory / state

In: [1, 2, 1, 0, 4, 5, 4, 5, 1, 2, 0, 1, 4, ...

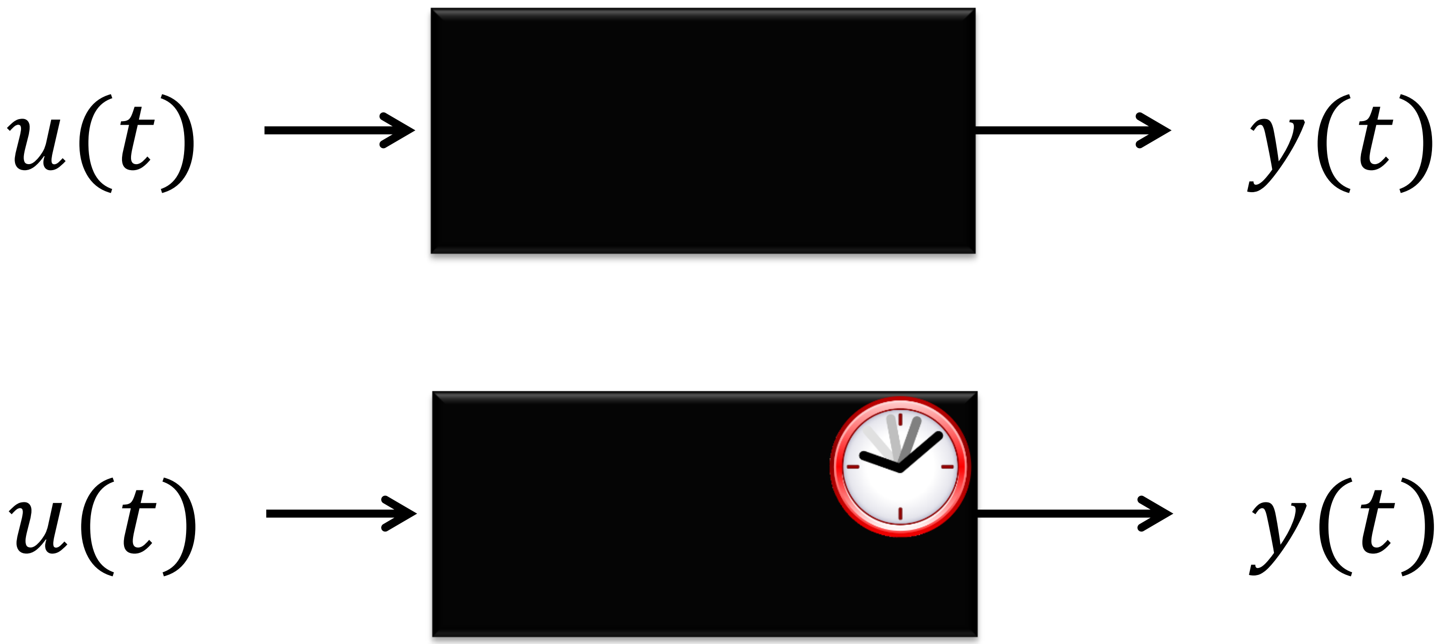
Out: [2, 1, 0, 4, 5, 4, 5, 1, 2, 0, 1, 4, ...

Time

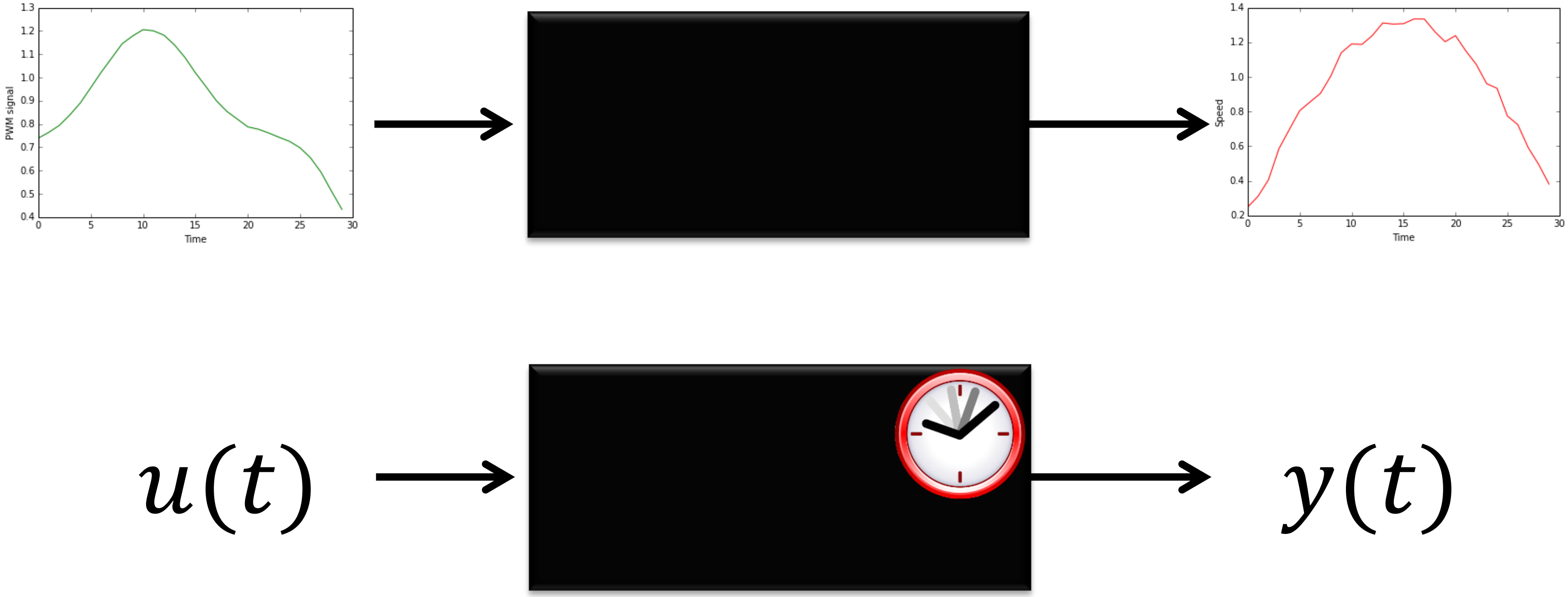
Time shift

Discrete **Deterministic** **Causal**

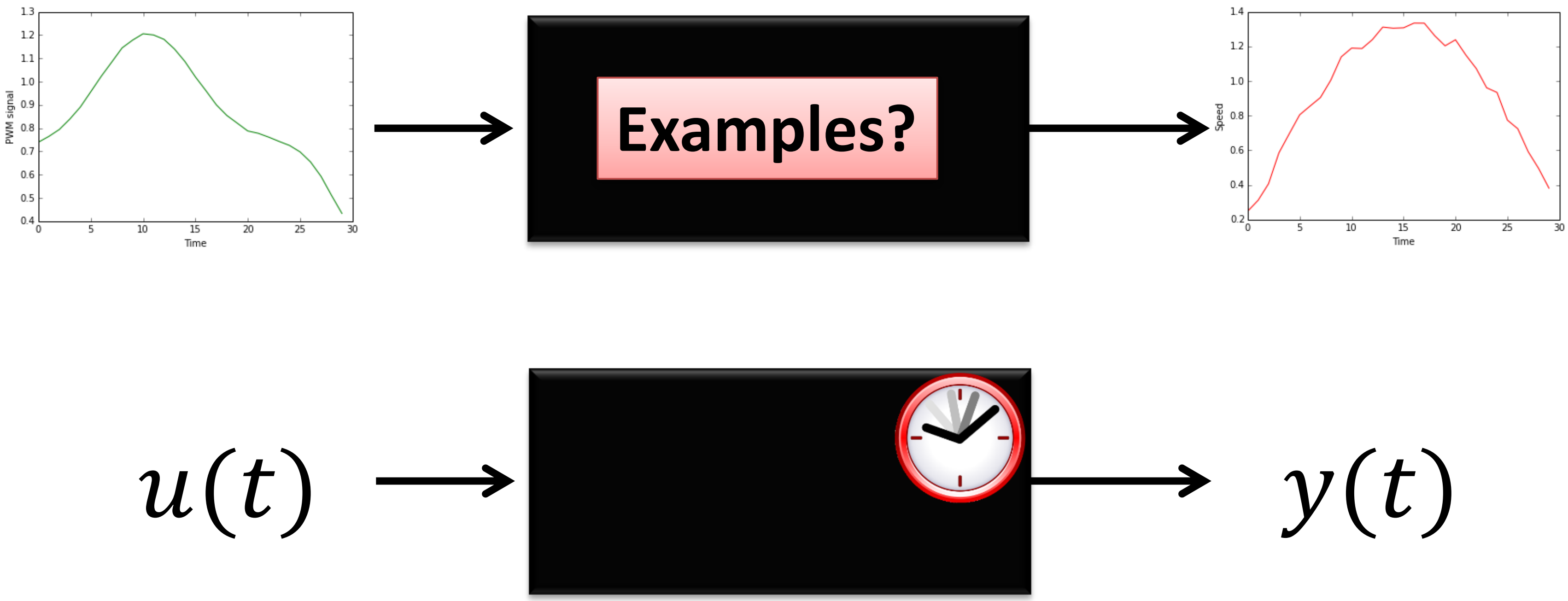
Time-invariant / Time-varying



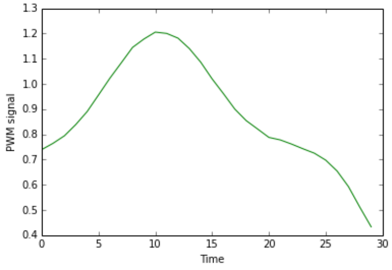
Time-invariant / Time-varying

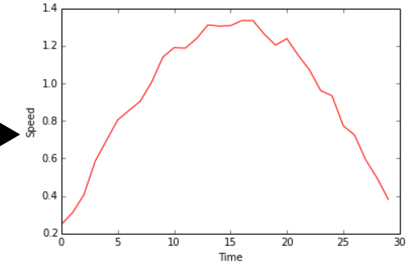


Time-invariant / Time-varying



Time-invariant / Time-varying



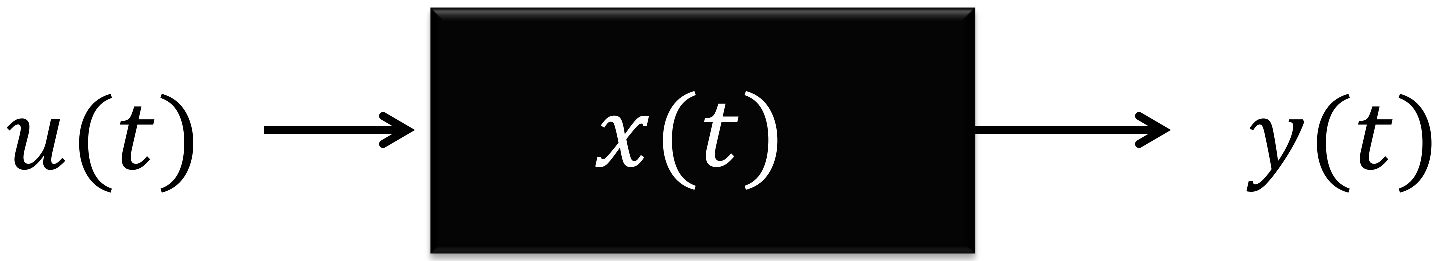


$u(t)$ → **Examples?** → $y(t)$

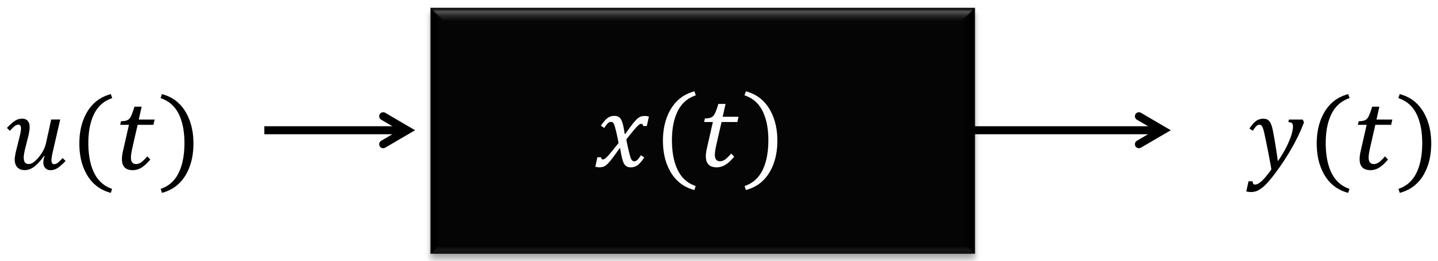
Discrete **Deterministic** **Causal**

Time-invariant

State-space model

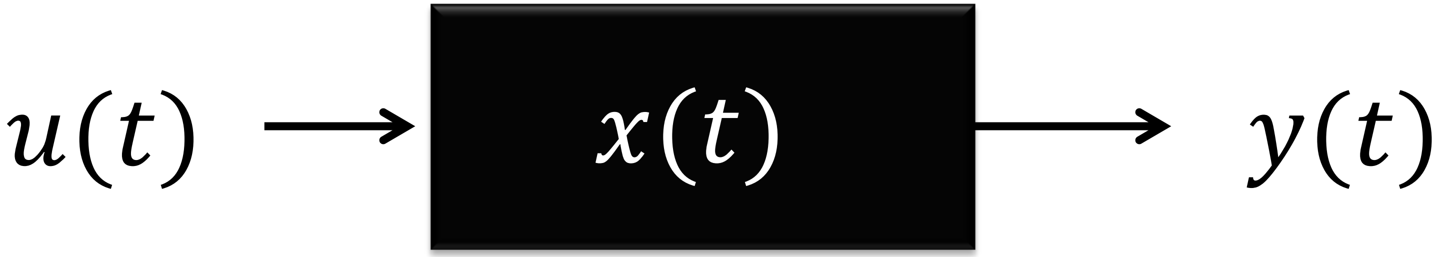


State-space model



Output and new state are determined by **the current state and the input.**

State-space model

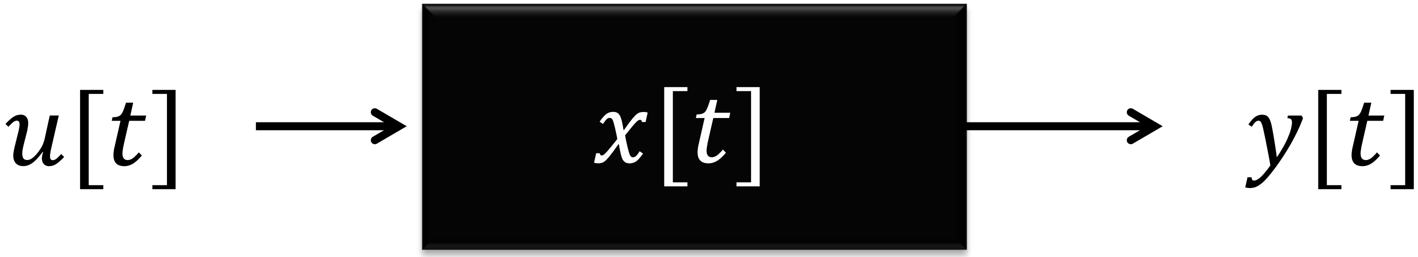


Output and new state are determined by **the current state and the input.**

$$x'(t) = \boldsymbol{F}\big(x(t), u(t)\big)$$

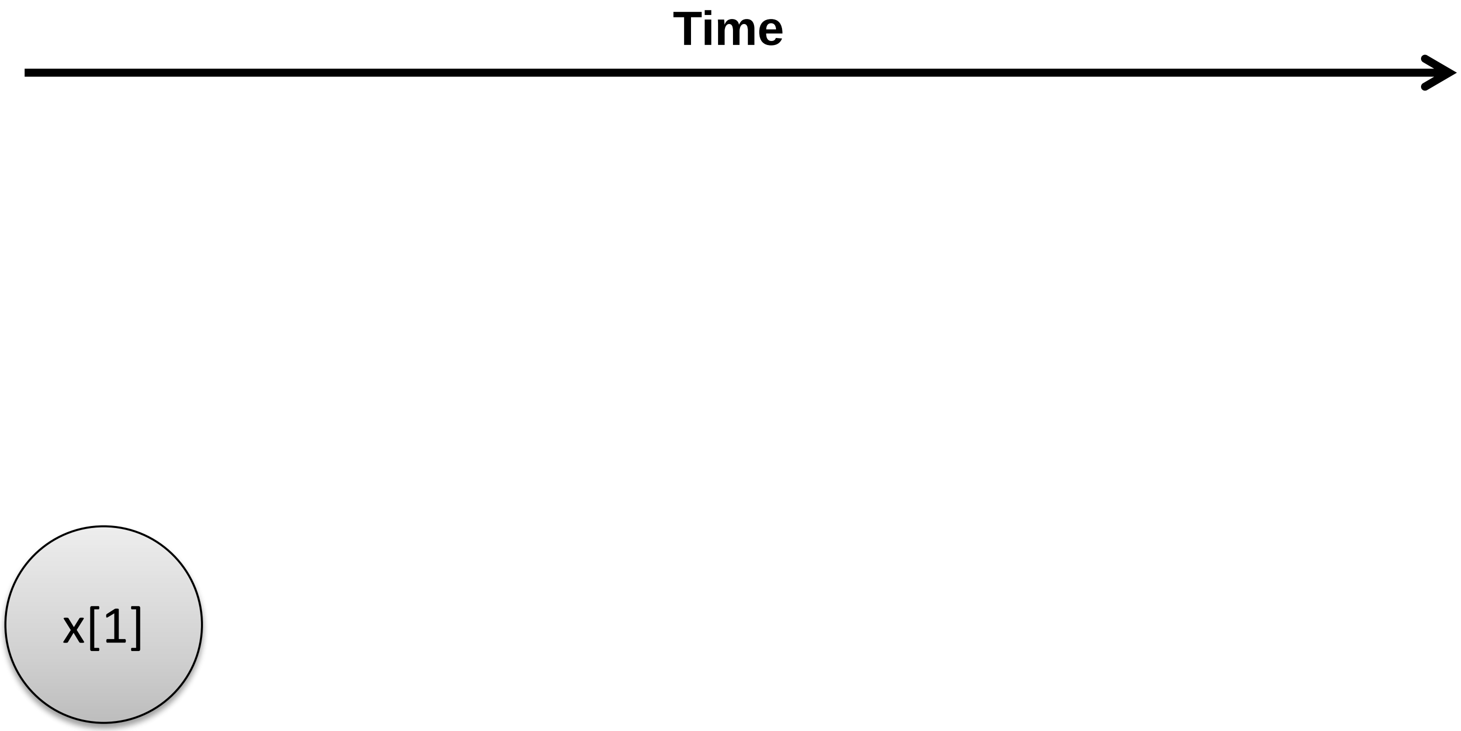
$$y(t) = \boldsymbol{G}(x(t), u(t))$$

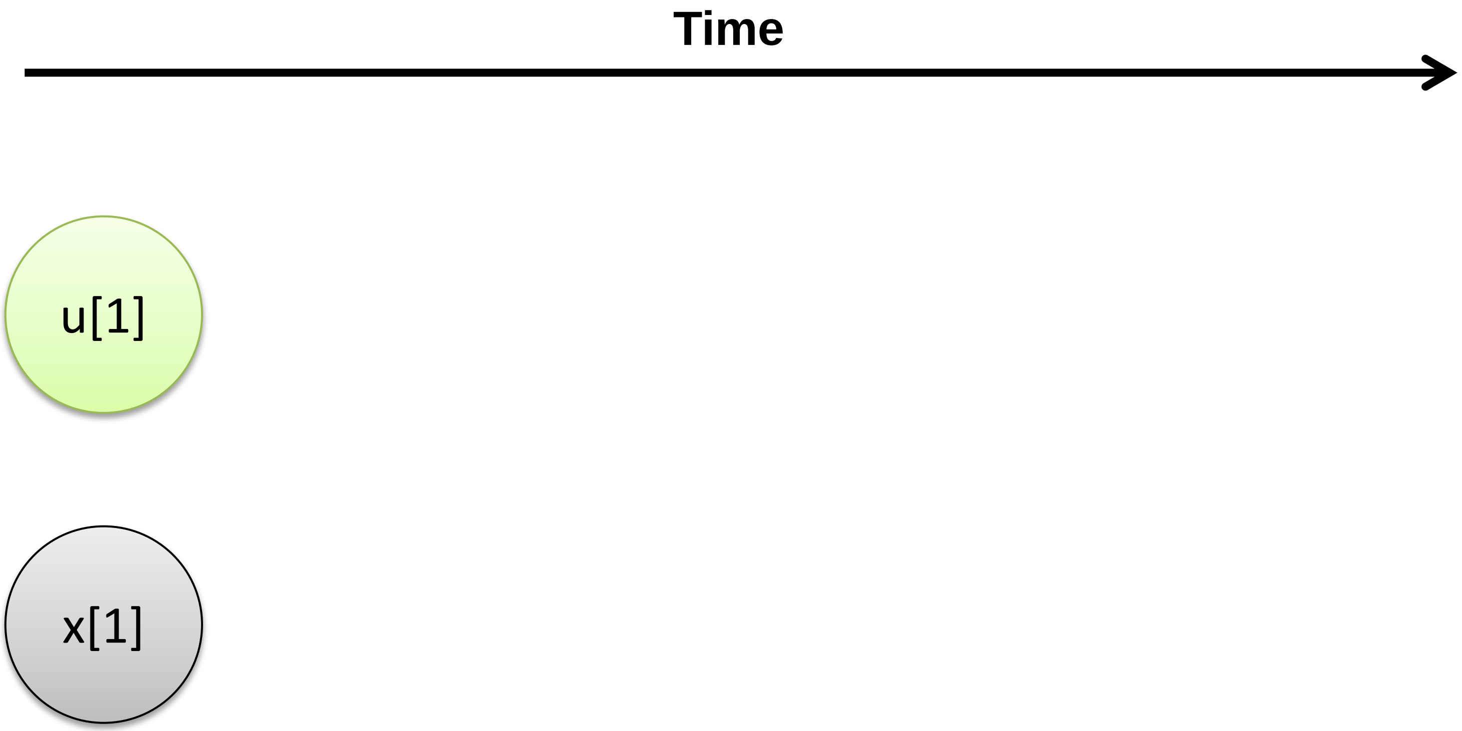
State-space model

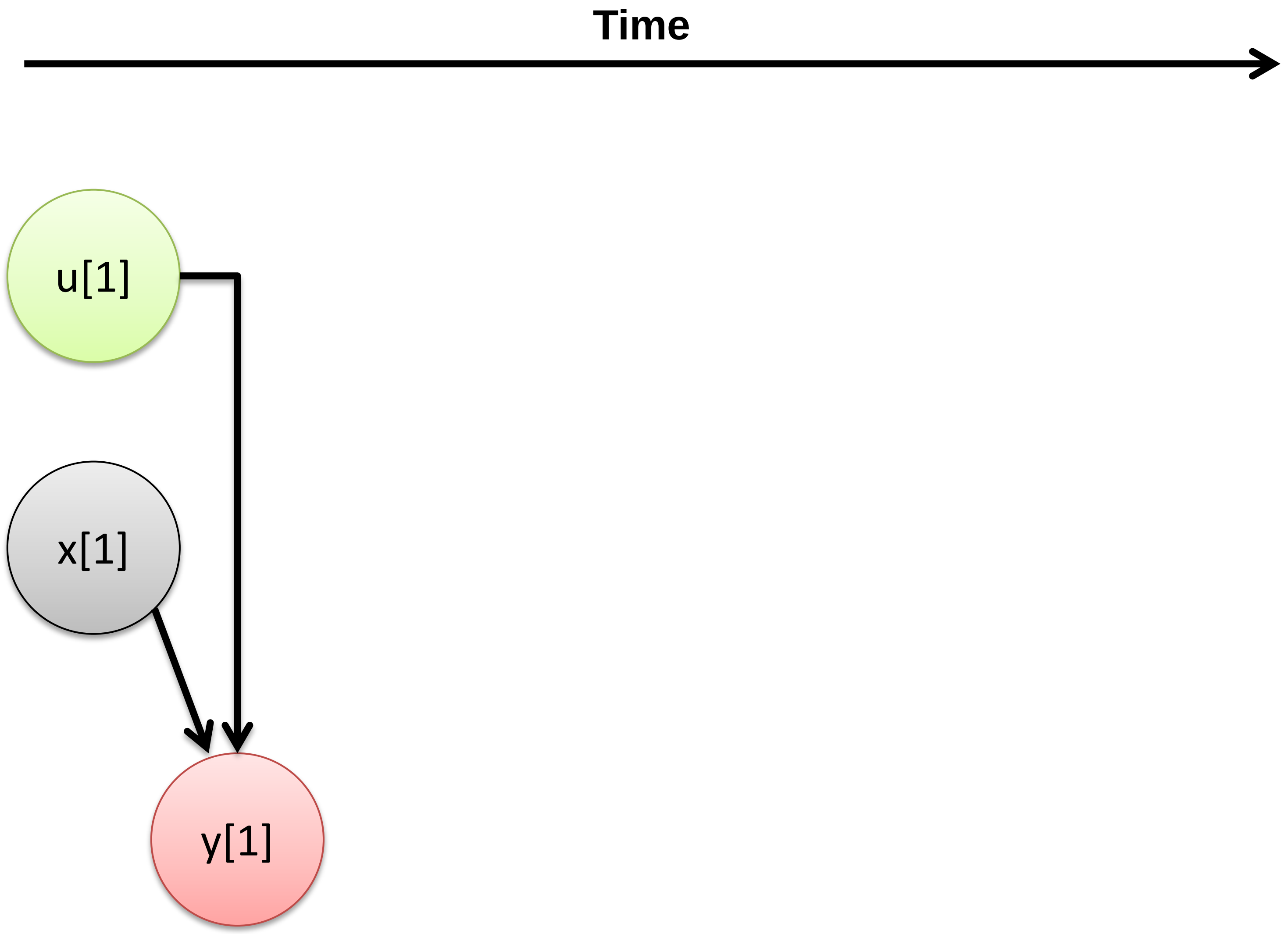


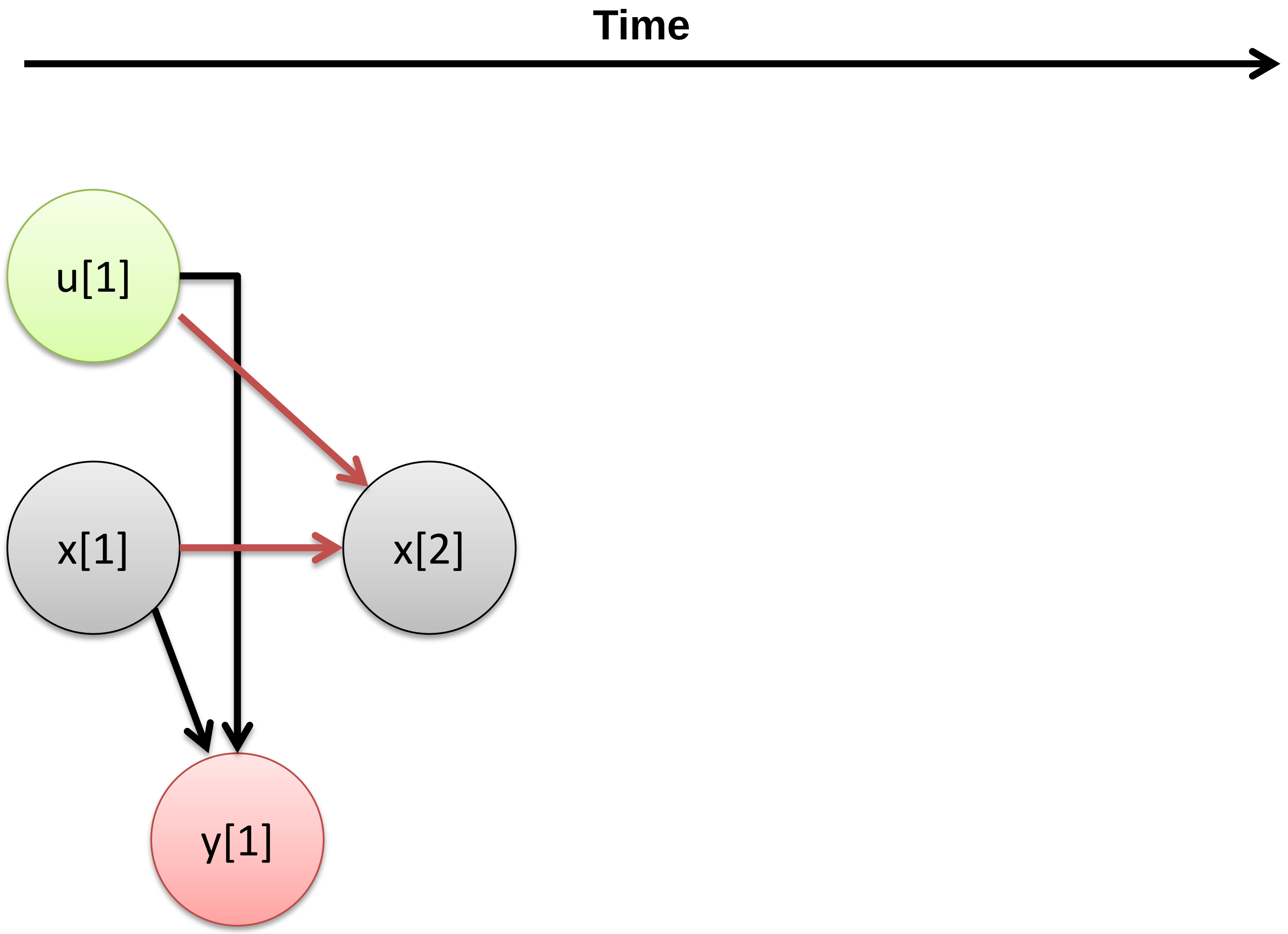
Output and new state are determined by **the current state and the input.**

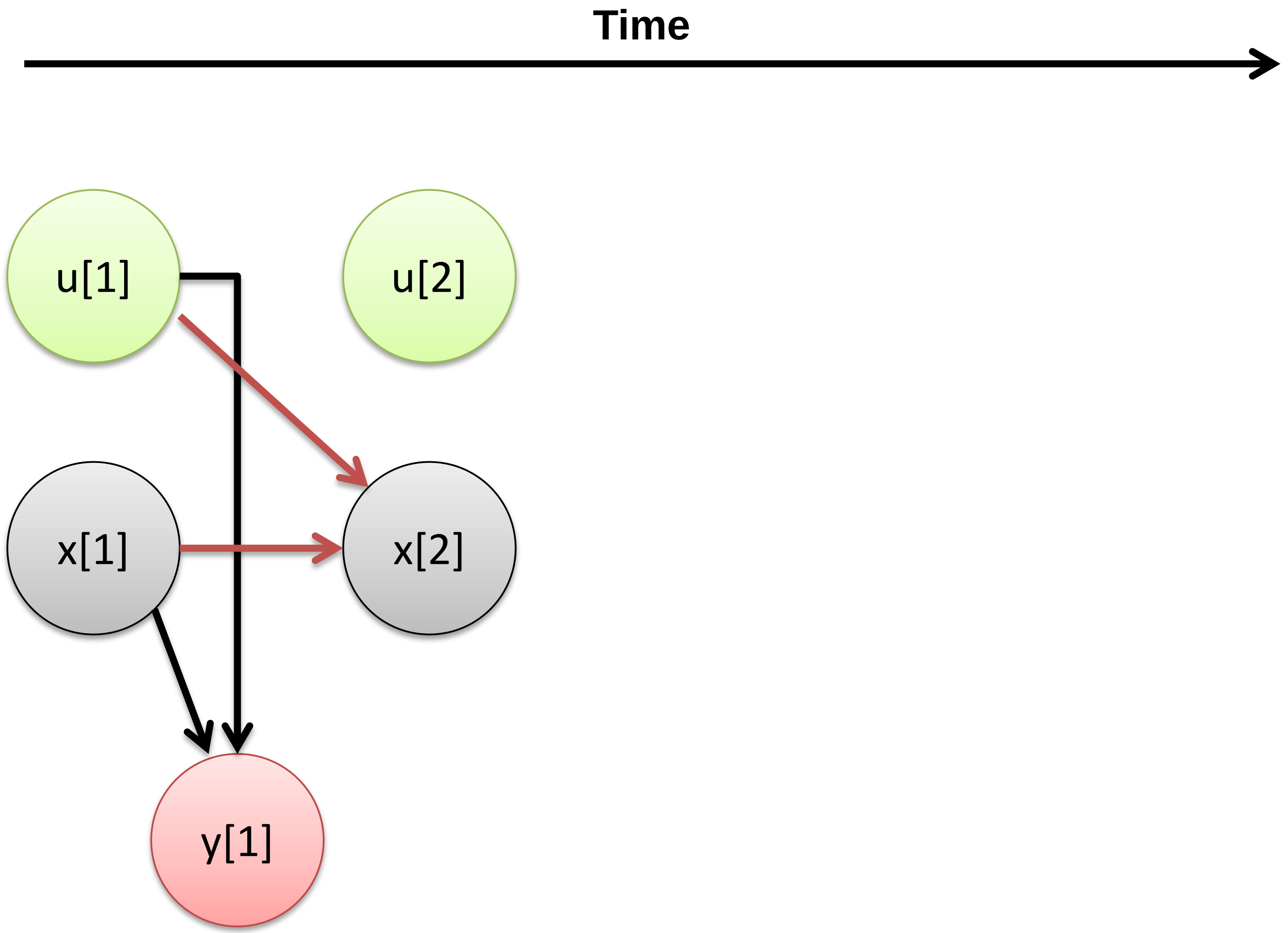
$$x[t+1] = \boldsymbol{F}(x[t], u[t])$$
$$y[t] = \boldsymbol{G}(x[t]\ u[t])$$


Time
x[1]

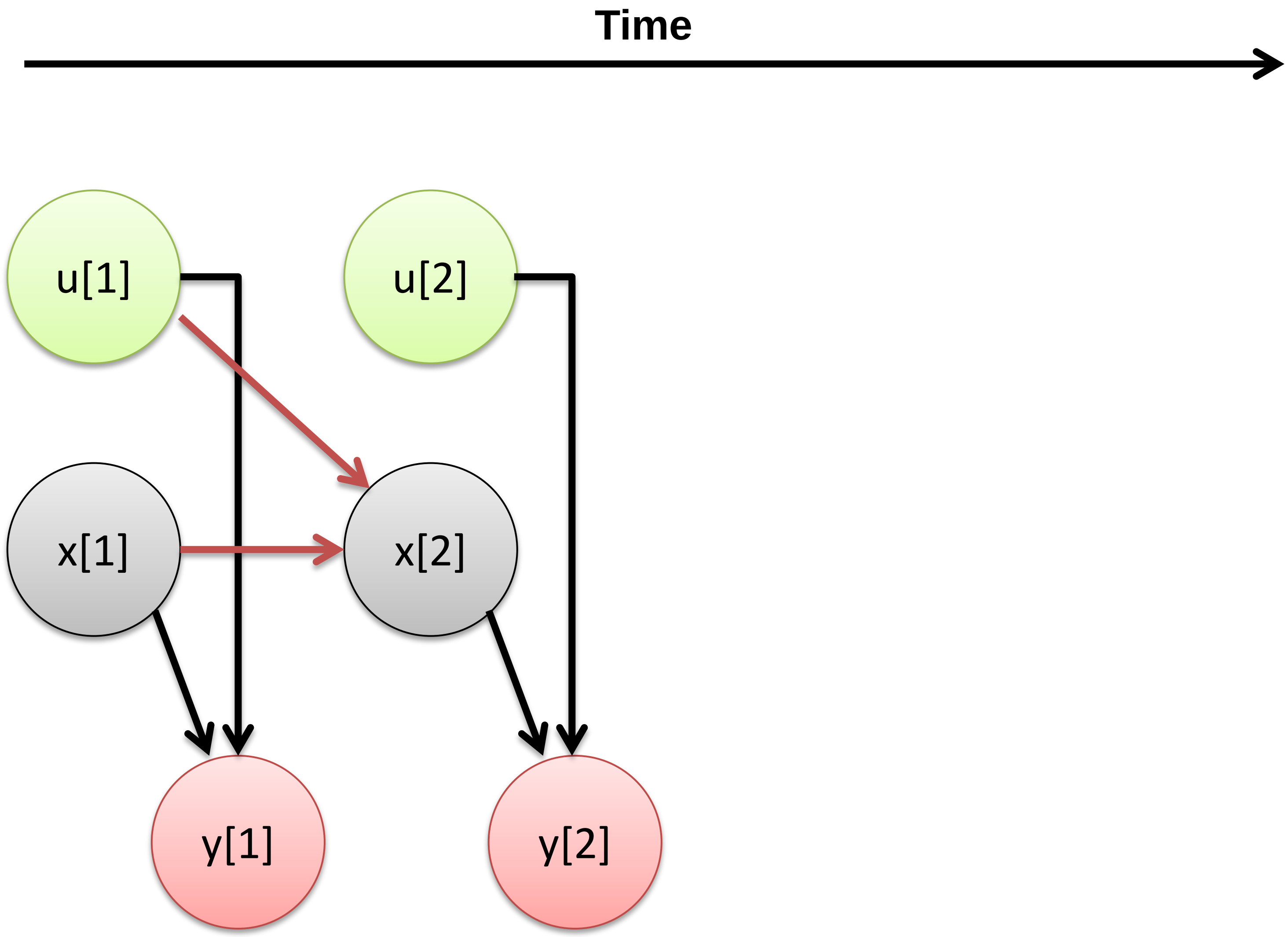

Time
u[1]
x[1]


Time
u[1]
x[1]
y[1]

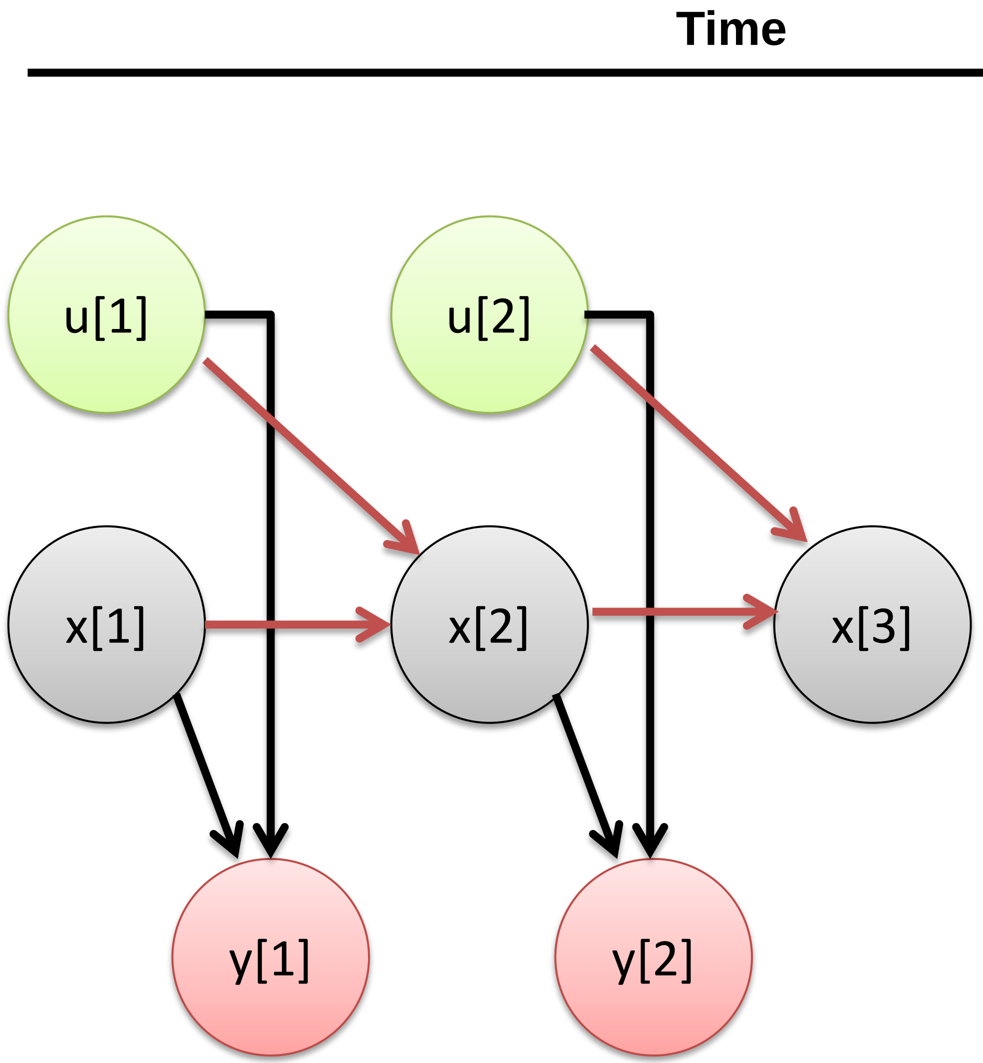

Time
u[1]
x[1]
x[2]
y[1]

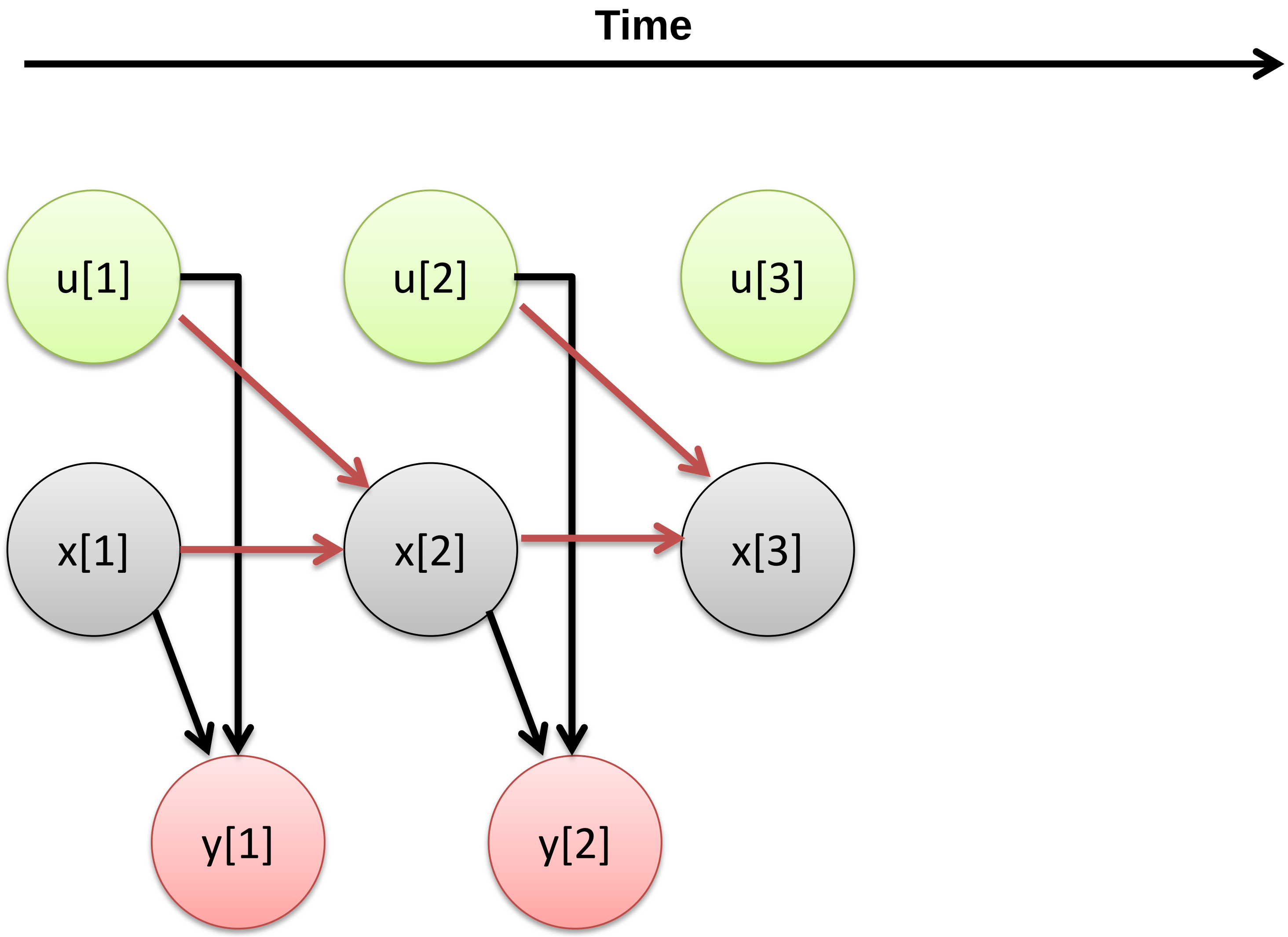


Time
u[1]
u[2]
x[1]
x[2]
y[1]

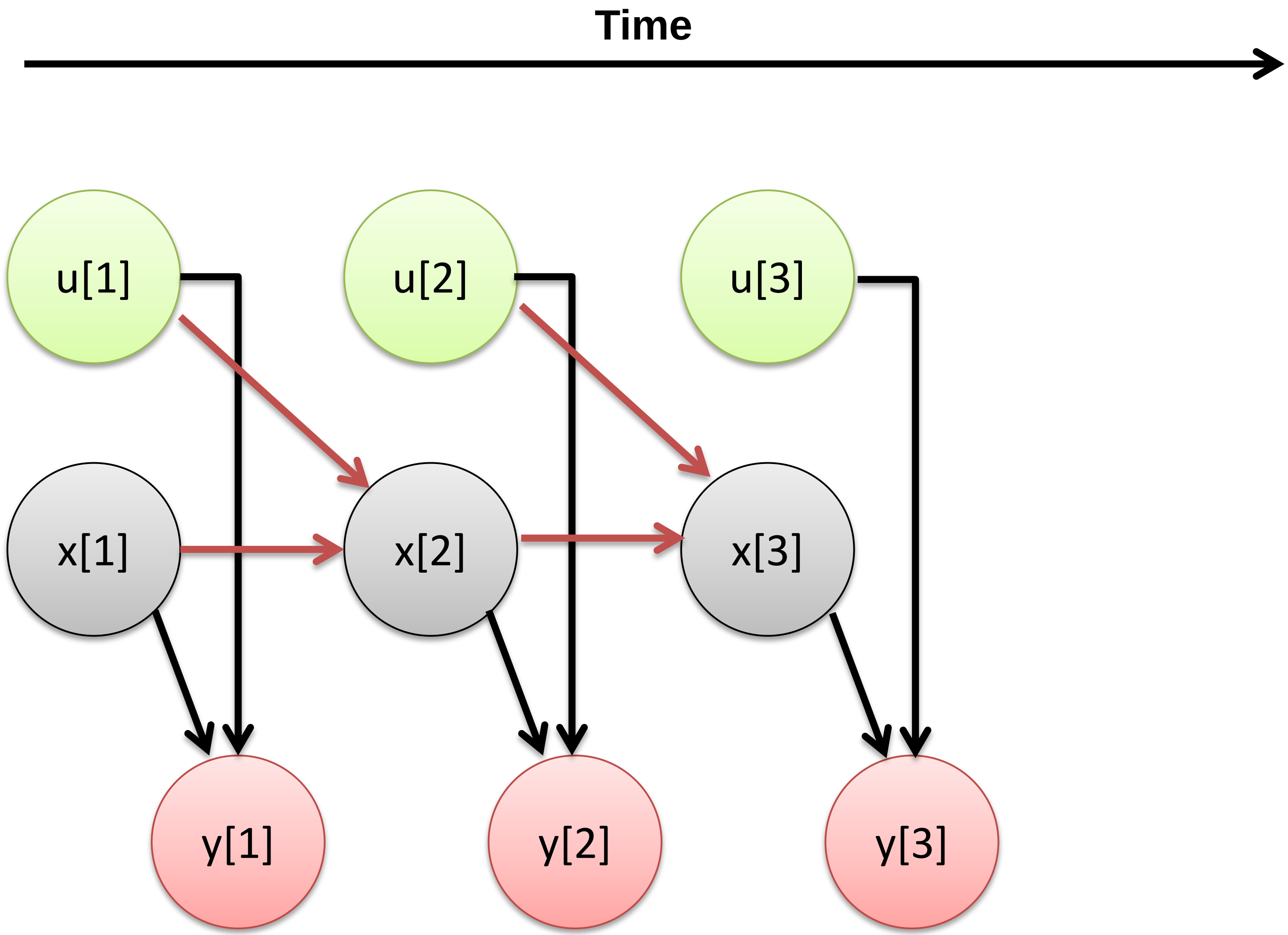


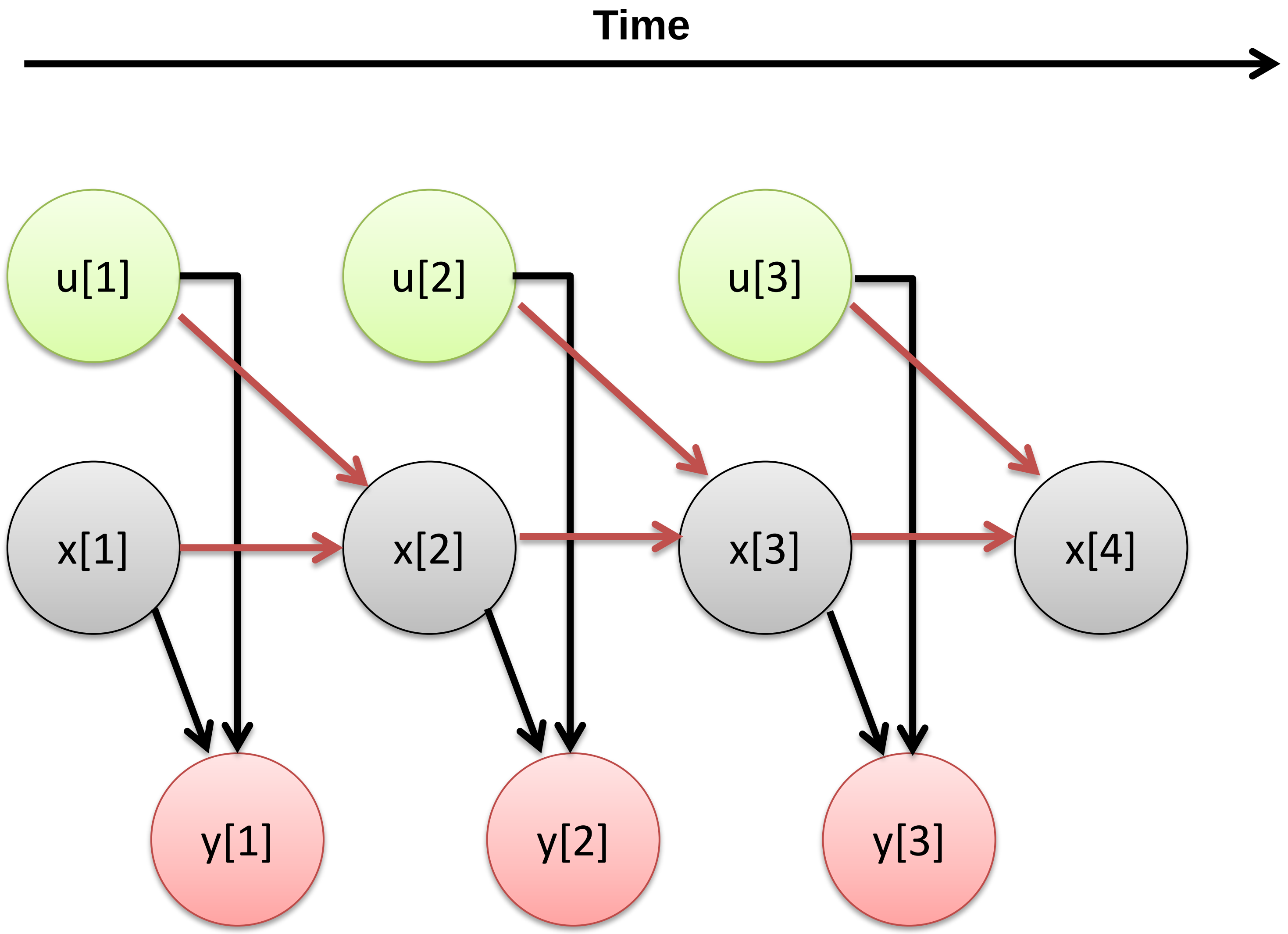
Time
u[1]
u[2]
x[1]
x[2]
y[1]
y[2]

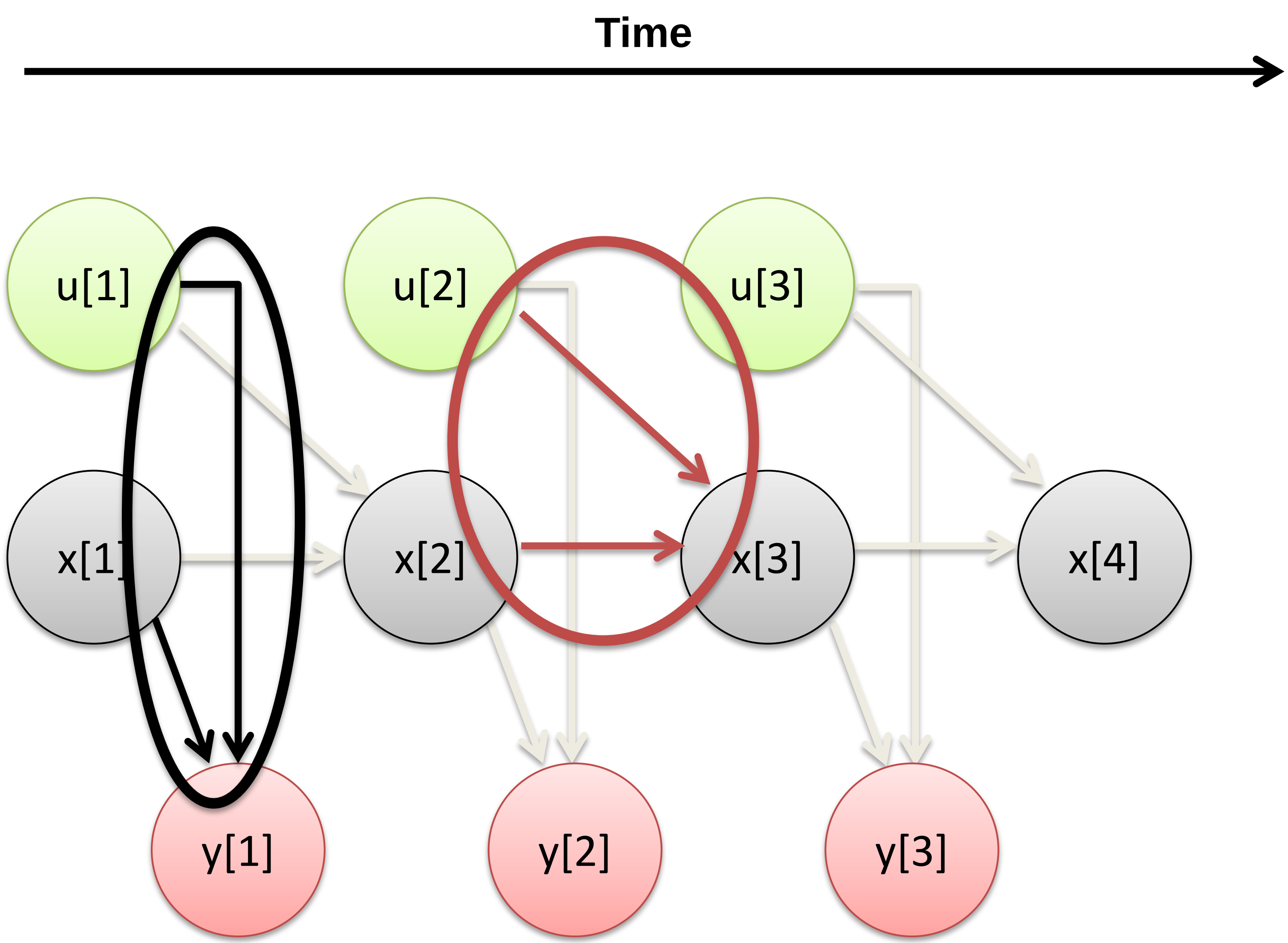

Time
u[1]
u[2]
x[1]
x[2]
x[3]
y[1]
y[2]



Time
u[1]
u[2]
u[3]
x[1]
x[2]
x[3]
y[1]
y[2]


Time
u[1]
u[2]
u[3]
x[1]
x[2]
x[3]
y[1]
y[2]
y[3]


Time
u[1]
u[2]
u[3]
x[1]
x[2]
x[3]
x[4]
y[1]
y[2]
y[3]



$$x[t+1] = \boldsymbol{F}(x[t], u[t])$$

$$y[t] = \boldsymbol{G}(x[t]\, u[t])$$

Dimensionality

- $x[t]$ can be
  - Finite-dimensional
  - Infinite-dimensional

Dimensionality

- $x[t]$ can be
  - Finite-dimensional
  - Infinite-dimensional

Examples?

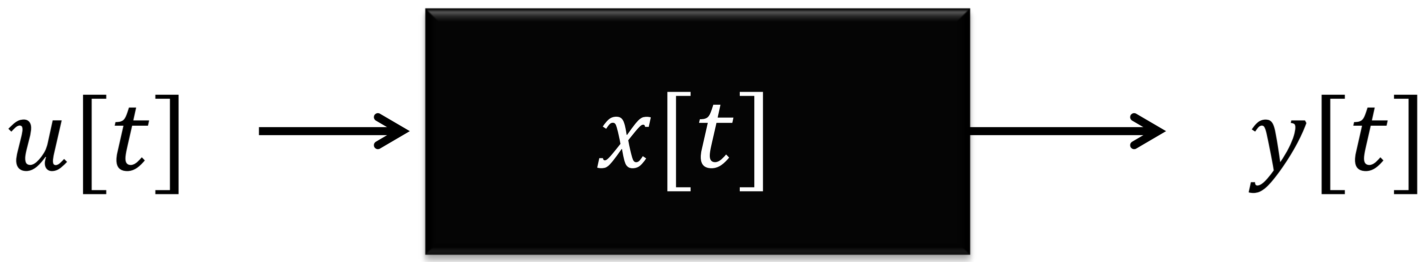
Dimensionality

- $x[t]$ can be
  - Finite-dimensional
  - Infinite-dimensional

Examples?

Continuous delay

$$y(t) = u(t-1)$$



Discrete

Deterministic

Causal

Time-invariant

Finite-dimensional

$$x[t+1] = \boldsymbol{F}(x[t], u[t])$$

$$y[t] = \boldsymbol{G}(x[t], u[t])$$

Discrete **Deterministic** **Causal**

Time-invariant **Finite-dimensional**

$$x[t+1] = \boldsymbol{A}x[t] + \boldsymbol{B}u[t]$$

$$y[t] = \boldsymbol{C}x[t] + \boldsymbol{D}u[t]$$

Discrete **Deterministic** **Causal**

Time-invariant **Finite-dimensional**

Linear

$$x'(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t)$$
$$y(t) = \boldsymbol{C}x(t) + \boldsymbol{D}u(t)$$

Continuous

Deterministic

Causal

Time-invariant

Finite-dimensional

Linear

$$x'(t) = \boldsymbol{A}x(t) + \boldsymbol{B}u(t)$$
$$y(t) = \boldsymbol{C}x(t) + \boldsymbol{D}u(t)$$

Example?

Continuous

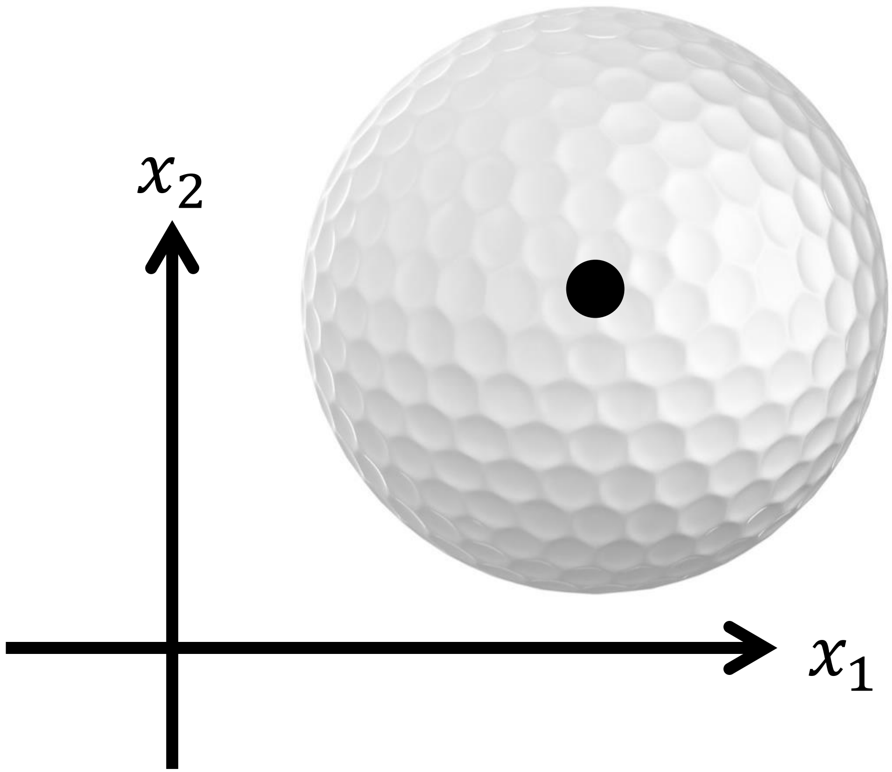
Deterministic

Causal

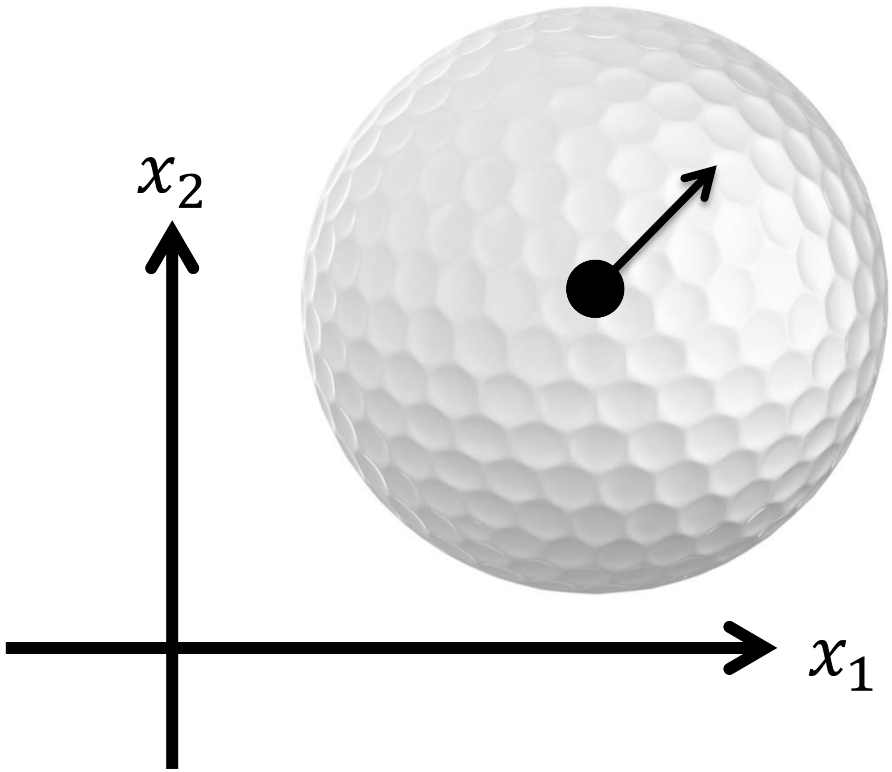
Time-invariant

Finite-dimensional

Linear

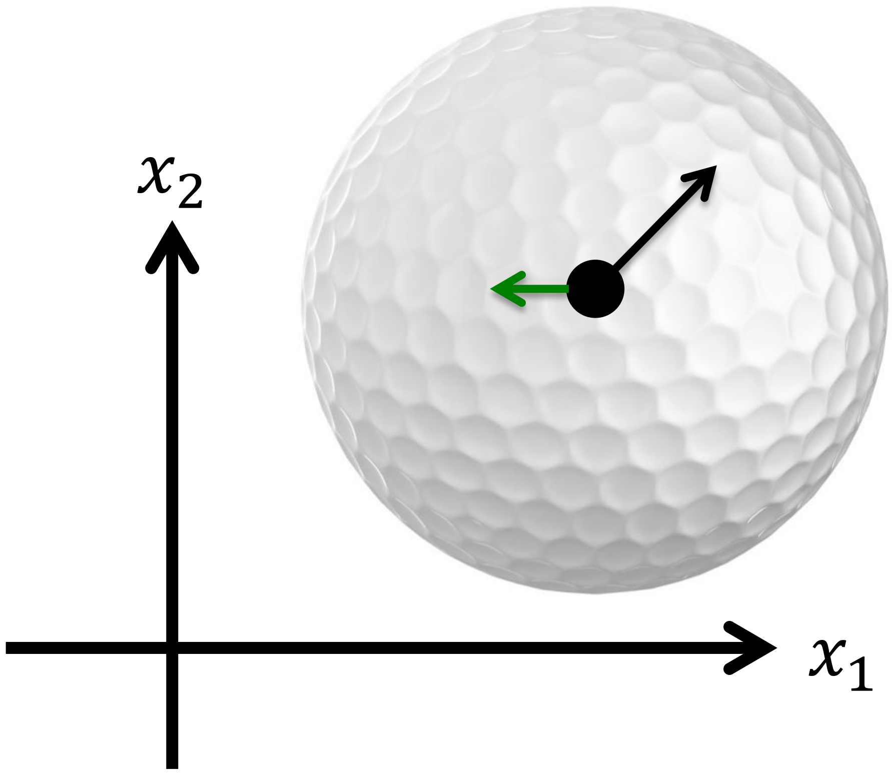


$\text{position} = (x_1(t), x_2(t))$



position $= (x_1(t), x_2(t))$

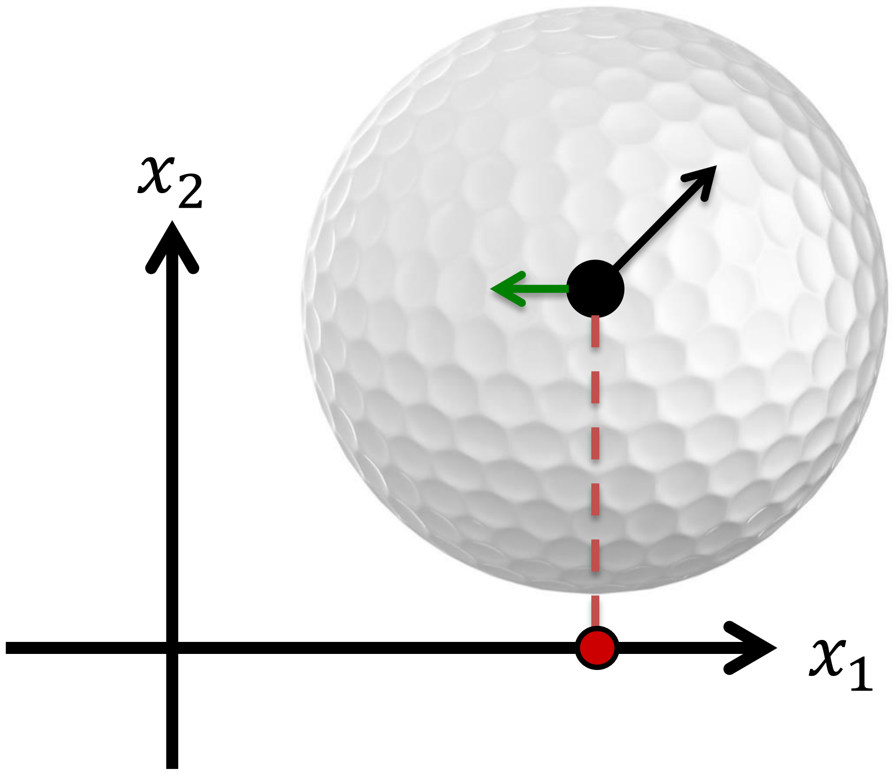
velocity $= (v_1(t), v_2(t))$



$$\text{position} = (x_1(t), x_2(t))$$

$$\text{velocity} = (v_1(t), v_2(t))$$

$$\text{force} = (u_1(t), u_2(t))$$

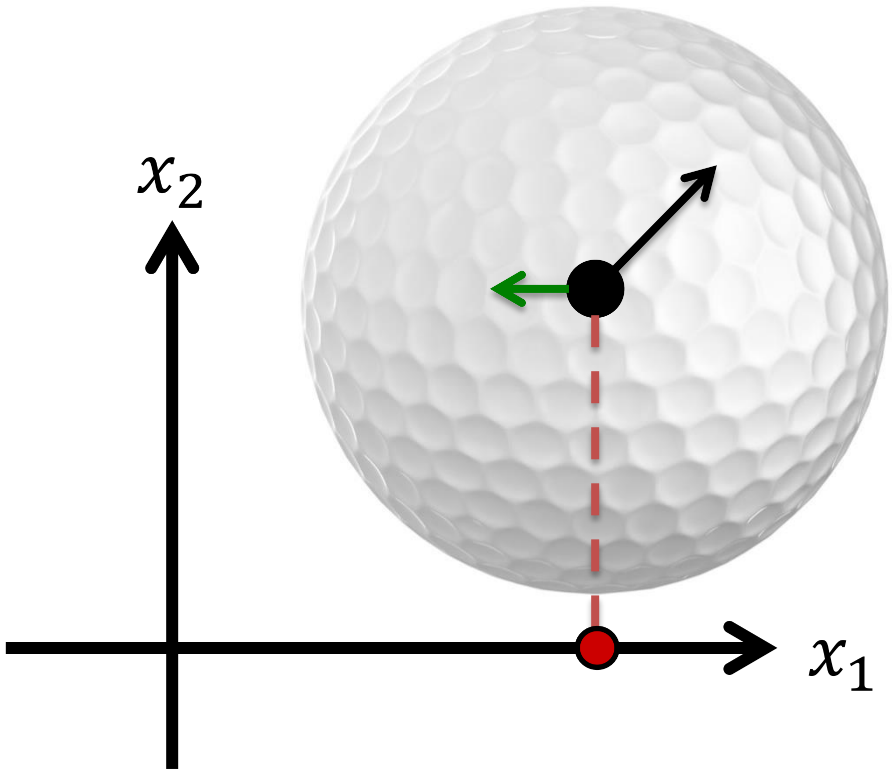


position $= (x_1(t), x_2(t))$

velocity $= (v_1(t), v_2(t))$

force $= (u_1(t), u_2(t))$

observe $= x_1(t)$

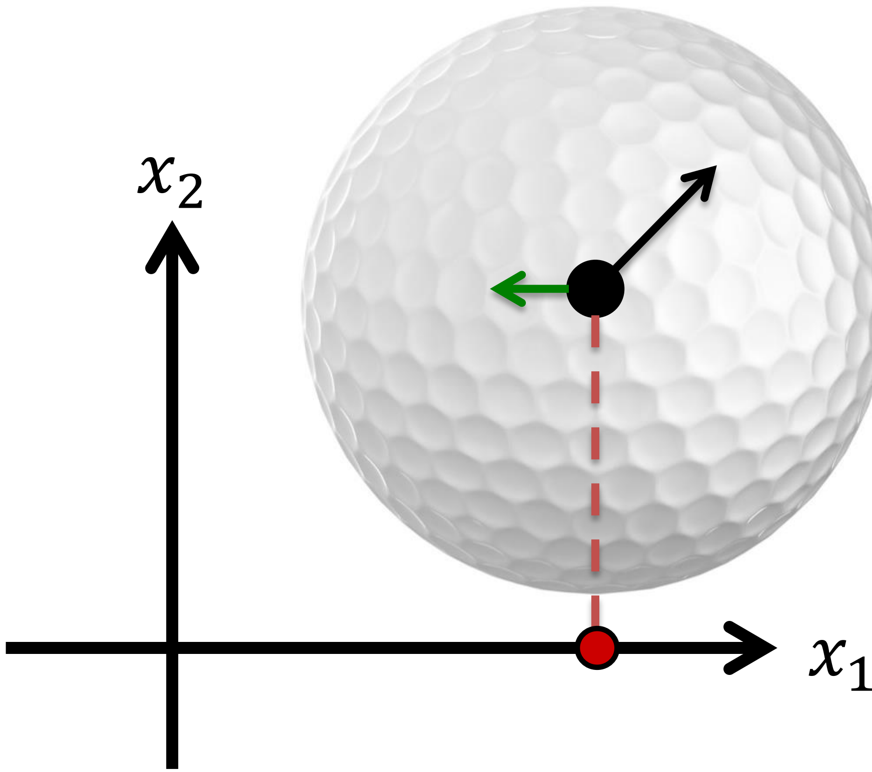


position $= (x_1(t), x_2(t))$

velocity $= (v_1(t), v_2(t))$

force $= (u_1(t), u_2(t))$

observe $= x_1(t)$



$$\boldsymbol{x}(t) = (x_1(t), x_2(t), v_1(t), v_2(t))$$

$$\boldsymbol{u}(t) = (u_1(t), u_2(t))$$

$$\boldsymbol{y}(t) = x_1(t)$$

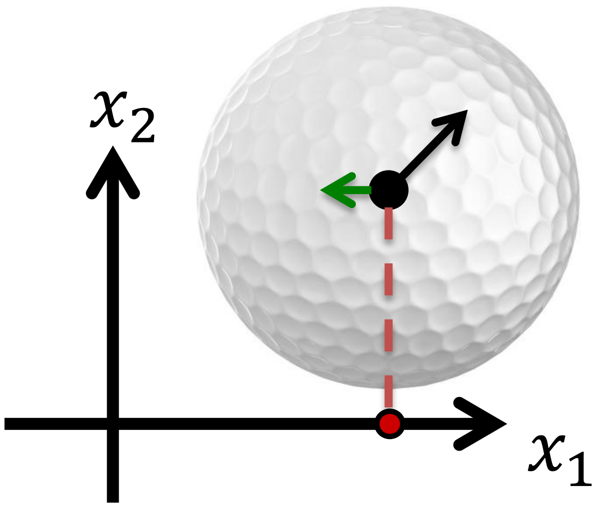
$$\boldsymbol{x}'(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t)$$

$$\boldsymbol{x}(t) = (x_1(t), x_2(t), v_1(t), v_2(t))$$

$$\boldsymbol{u}(t) = (u_1(t), u_2(t))$$

$$\boldsymbol{y}(t) = x_1(t)$$



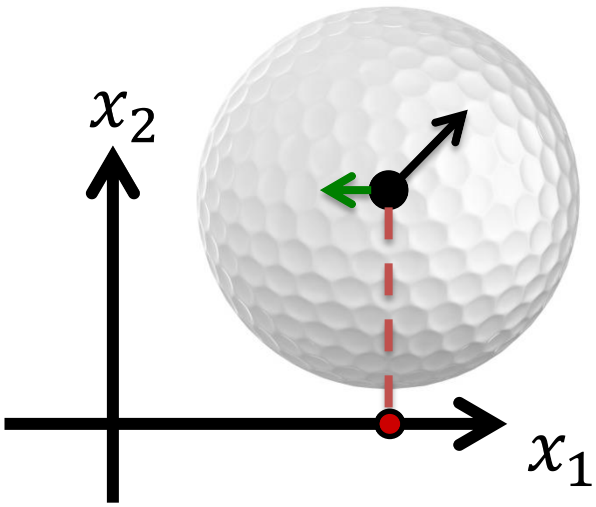
$$x_1'(t) =$$
$$x_2'(t) =$$
$$v_1'(t) =$$
$$v_2'(t) =$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t)$$

$$\boldsymbol{x}(t) = (x_1(t), x_2(t), v_1(t), v_2(t))$$

$$\boldsymbol{u}(t) = (u_1(t), u_2(t))$$

$$\boldsymbol{y}(t) = x_1(t)$$



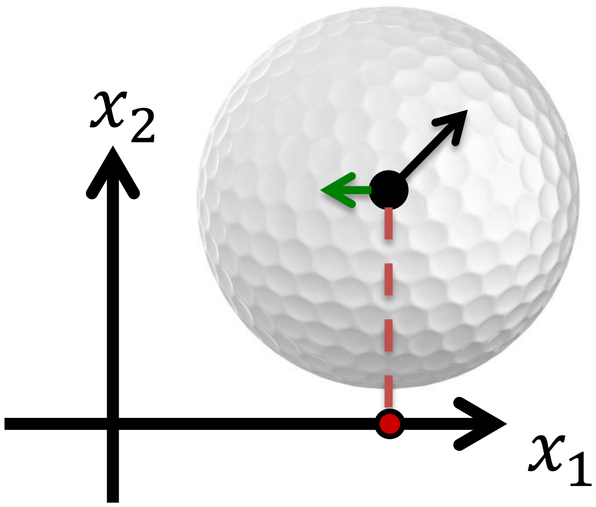
$$x_1'(t) = v_1(t)$$
$$x_2'(t) = v_2(t)$$
$$v_1'(t) =$$
$$v_2'(t) =$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t)$$

$$\boldsymbol{x}(t) = (x_1(t), x_2(t), v_1(t), v_2(t))$$

$$\boldsymbol{u}(t) = (u_1(t), u_2(t))$$

$$\boldsymbol{y}(t) = x_1(t)$$



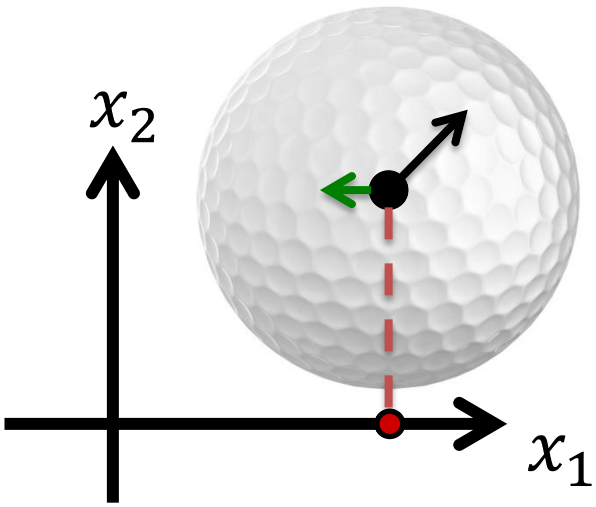
$$x_1'(t) = v_1(t)$$
$$x_2'(t) = v_2(t)$$
$$v_1'(t) = u_1(t)/M$$
$$v_2'(t) = u_2(t)/M$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t)$$

$$\boldsymbol{x}(t) = (x_1(t), x_2(t), v_1(t), v_2(t))$$

$$\boldsymbol{u}(t) = (u_1(t), u_2(t))$$

$$\boldsymbol{y}(t) = x_1(t)$$

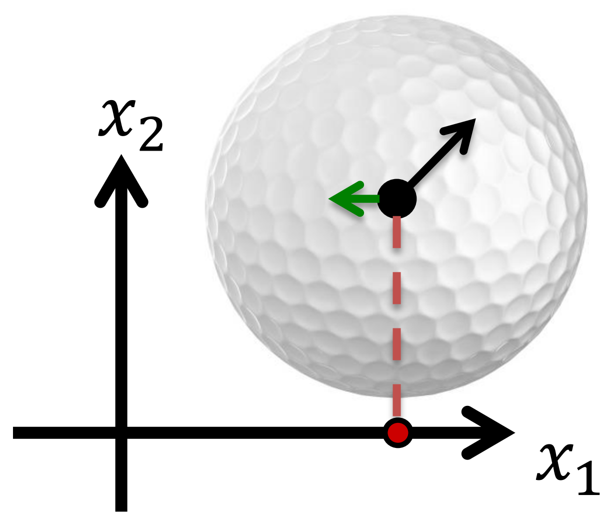


$$\begin{pmatrix} x_1{}' \\ x_2{}' \\ v_1{}' \\ v_2{}' \end{pmatrix} = \begin{pmatrix} \quad & \quad & \quad & \quad \\ & & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{pmatrix} + \begin{pmatrix} \quad & \quad \\ & \\ & \\ & \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\boldsymbol{u}(t) = (u_1(t), u_2(t))$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t)$$

$$\boldsymbol{y}(t) = x_1(t)$$

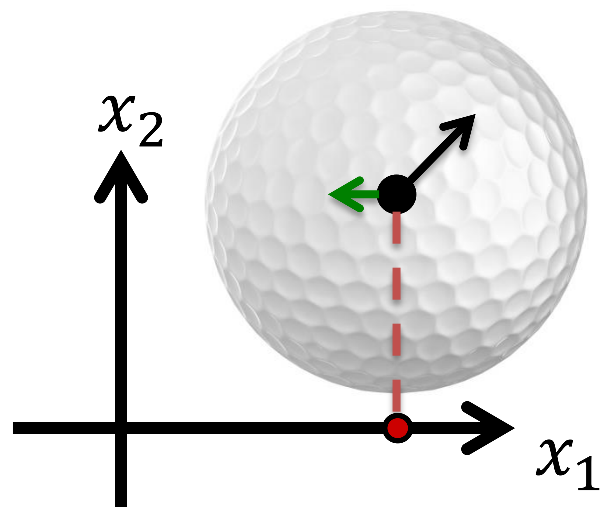


$$\begin{pmatrix} x_1' \\ x_2' \\ v_1' \\ v_2' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ & & & \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ & \\ & \\ & \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\boldsymbol{u}(t) = (u_1(t), u_2(t))$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t)$$

$$\boldsymbol{y}(t) = x_1(t)$$

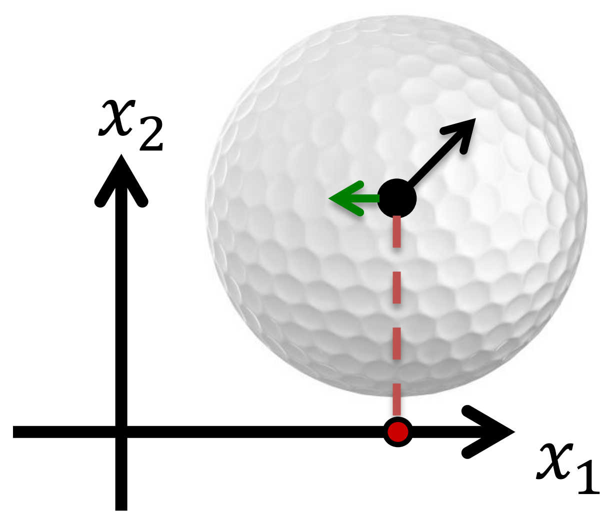


$$\begin{pmatrix} x_1' \\ x_2' \\ v_1' \\ v_2' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ & & & \\ & & & \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ & \\ & \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\boldsymbol{u}(t) = (u_1(t), u_2(t))$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t)$$

$$\boldsymbol{y}(t) = x_1(t)$$

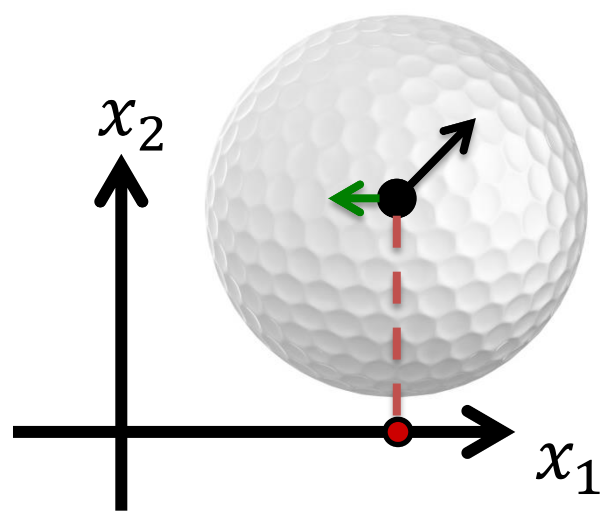


$$\begin{pmatrix} x_1' \\ x_2' \\ v_1' \\ v_2' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1/M & 0 \\ 0 & 1/M \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\boldsymbol{u}(t) = (u_1(t), u_2(t))$$

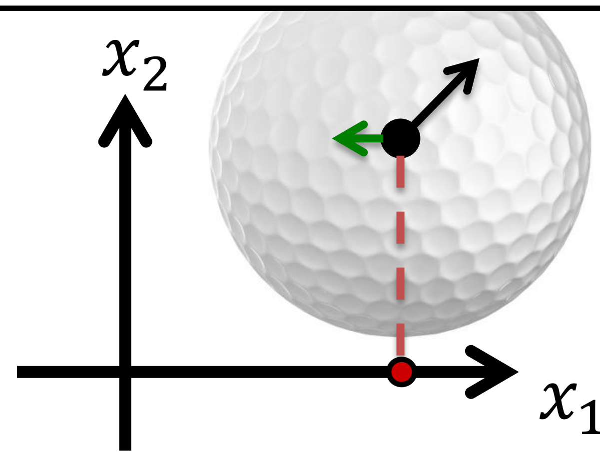
$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{x}(t) + \boldsymbol{D}\boldsymbol{u}(t)$$

$$\boldsymbol{y}(t) = x_1(t)$$



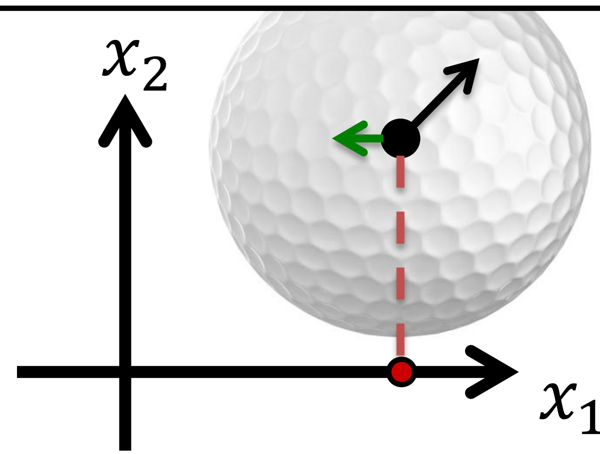
$$\begin{pmatrix} x_1' \\ x_2' \\ v_1' \\ v_2' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1/M & 0 \\ 0 & 1/M \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$(x_1) = (\qquad\qquad\qquad\qquad) \begin{pmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{pmatrix} + (\qquad\qquad) \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$



$$\begin{pmatrix} x_1' \\ x_2' \\ v_1' \\ v_2' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1/M & 0 \\ 0 & 1/M \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

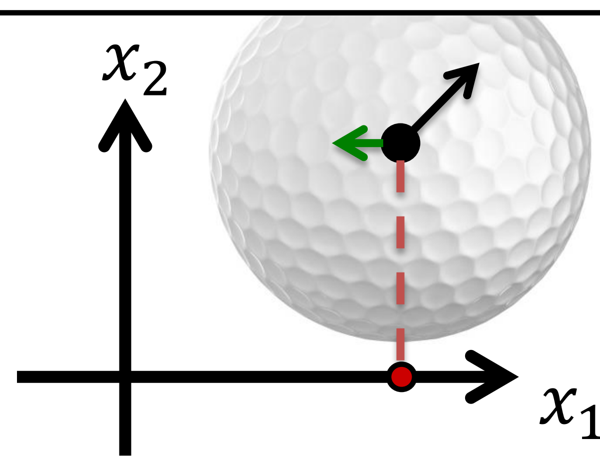
$$(x_1) = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$



$$\begin{pmatrix} x_1' \\ x_2' \\ v_1' \\ v_2' \end{pmatrix} = \begin{pmatrix} & & \mathbf{A} & & \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{pmatrix} + \begin{pmatrix} & \mathbf{B} & \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$u(t) = (u_1(t), u_2(t))$

$$(x_1) = (\quad \mathbf{C} \quad) \begin{pmatrix} x_1 \\ x_2 \\ v_1 \\ v_2 \end{pmatrix} + (\mathbf{D}) \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$



$$x[t+1] = \boldsymbol{A}x[t] + \boldsymbol{B}u[t]$$
$$y[t] = \boldsymbol{C}x[t] + \boldsymbol{D}u[t]$$

Discrete

Deterministic

Causal

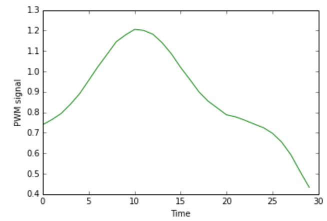
Time-invariant

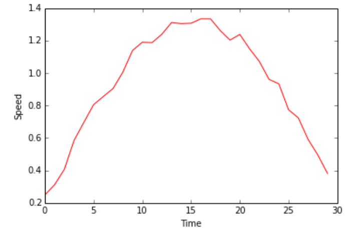
Finite-dimensional

Linear

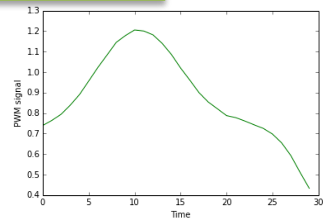
Dynamic system

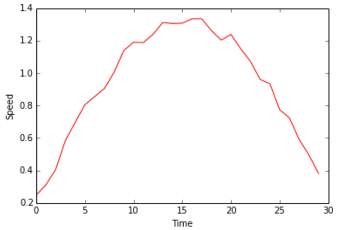
$(\boldsymbol{u}_\mathbf{1}[0], \boldsymbol{u}_\mathbf{1}[1], \ldots)$ → $(\boldsymbol{y}_\mathbf{1}[0], \boldsymbol{y}_\mathbf{1}[1], \ldots)$

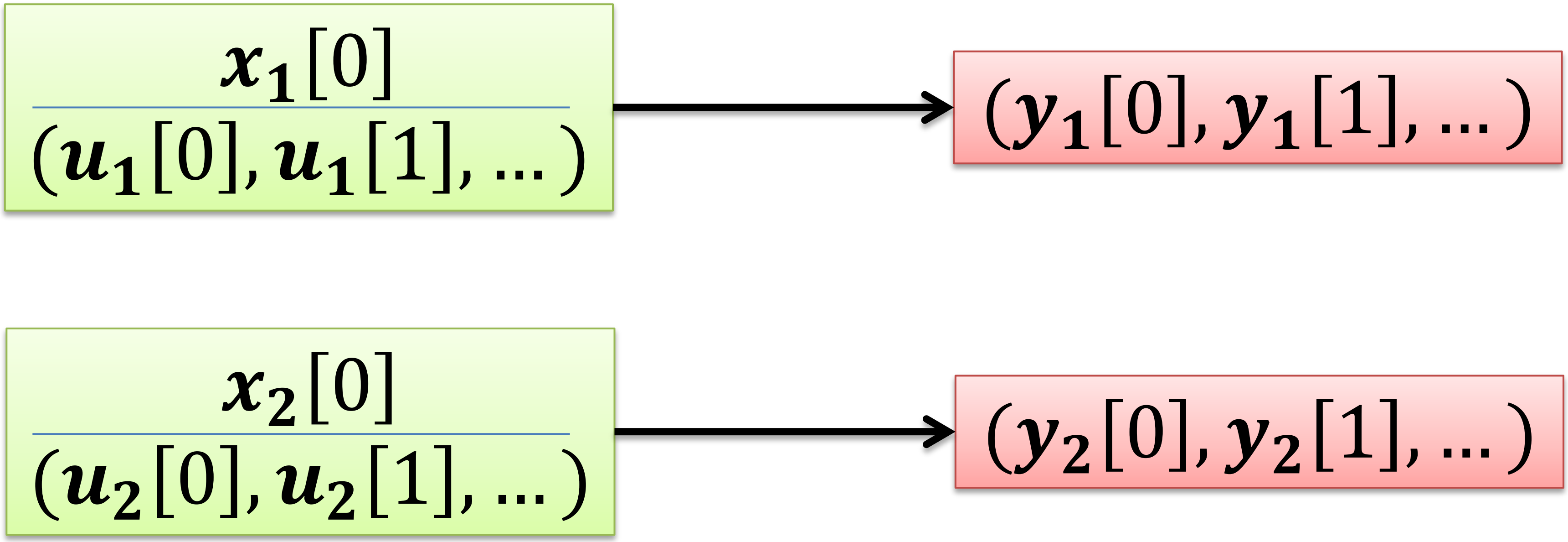


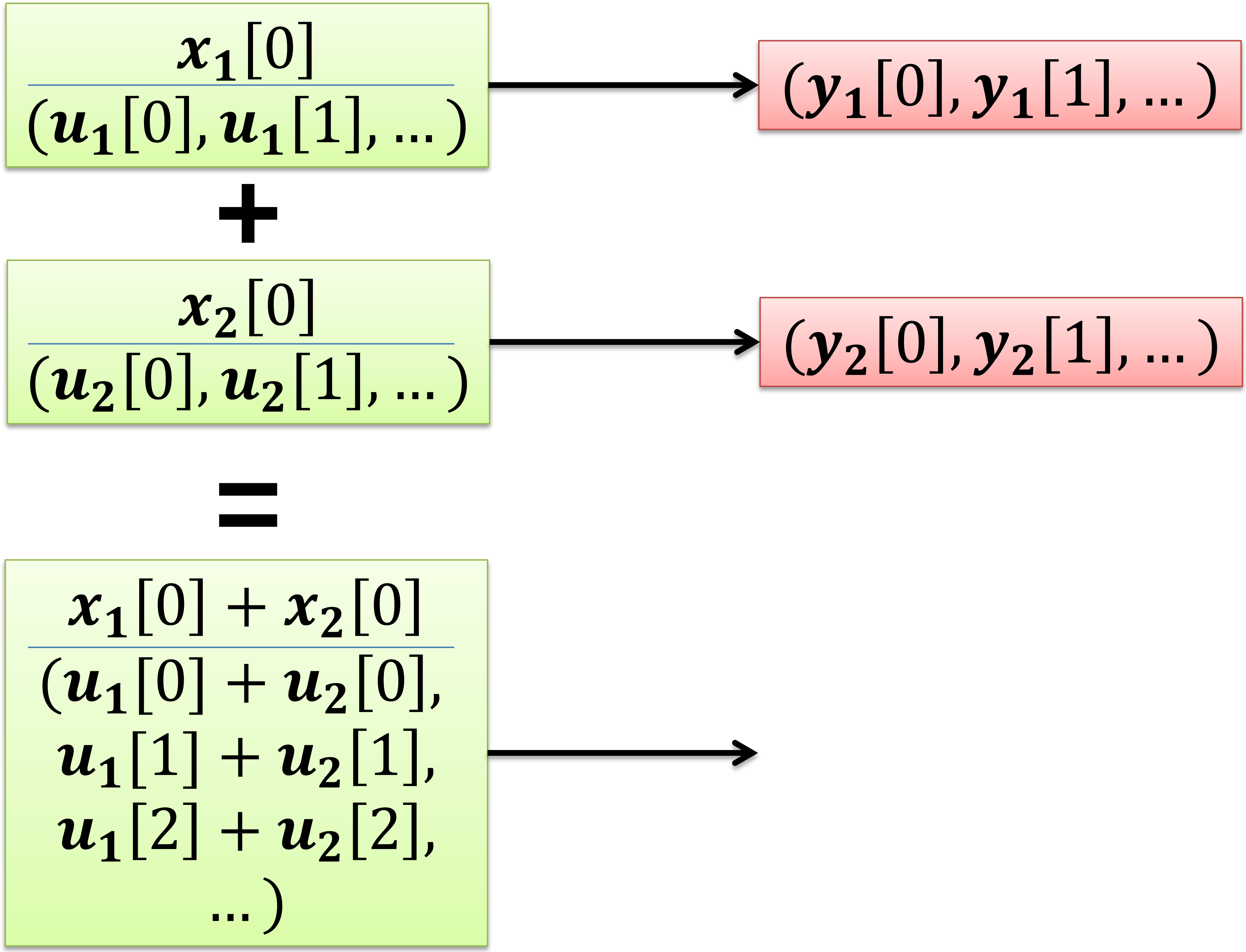


$$\frac{\boldsymbol{x}_1[0]}{(\boldsymbol{u}_1[0], \boldsymbol{u}_1[1], \dots)} \rightarrow (\boldsymbol{y}_1[0], \boldsymbol{y}_1[1], \dots)$$

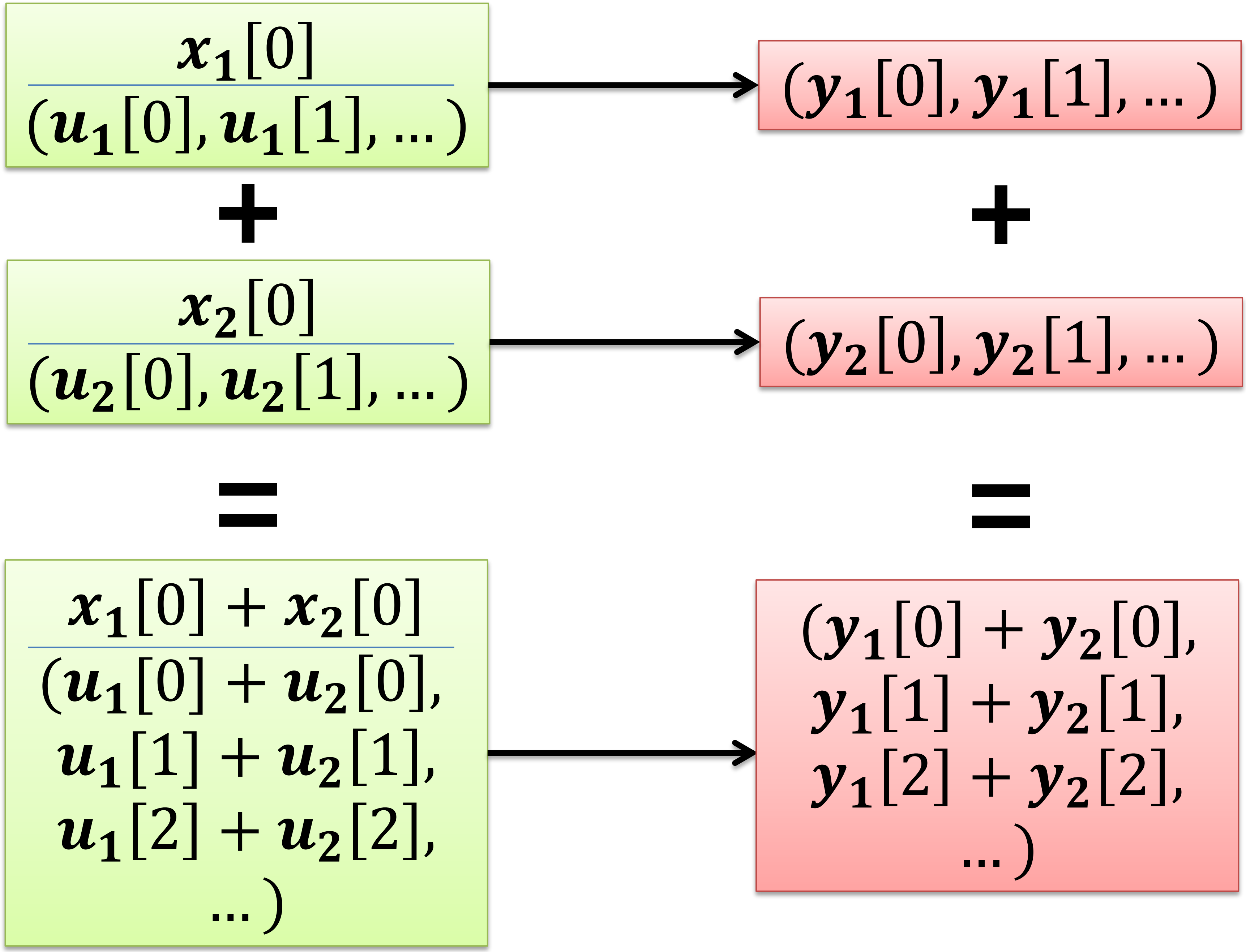





$\frac{x_1[0]}{(u_1[0], u_1[1], \dots)}$
$(y_1[0], y_1[1], \dots)$
$\frac{x_2[0]}{(u_2[0], u_2[1], \dots)}$
$(y_2[0], y_2[1], \dots)$



$\frac{x_1[0]}{(u_1[0], u_1[1], \ldots)}$
$(y_1[0], y_1[1], \ldots)$
+
$\frac{x_2[0]}{(u_2[0], u_2[1], \ldots)}$
$(y_2[0], y_2[1], \ldots)$
=
$\frac{x_1[0] + x_2[0]}{(u_1[0] + u_2[0], u_1[1] + u_2[1], u_1[2] + u_2[2], \ldots)}$



$\frac{x_1[0]}{(u_1[0], u_1[1], \dots)}$
$(y_1[0], y_1[1], \dots)$
+
+
$\frac{x_2[0]}{(u_2[0], u_2[1], \dots)}$
$(y_2[0], y_2[1], \dots)$
=
=
$\frac{x_1[0] + x_2[0]}{(u_1[0] + u_2[0], u_1[1] + u_2[1], u_1[2] + u_2[2], \dots)}$
$(y_1[0] + y_2[0], y_1[1] + y_2[1], y_1[2] + y_2[2], \dots)$

Linearity

+ +

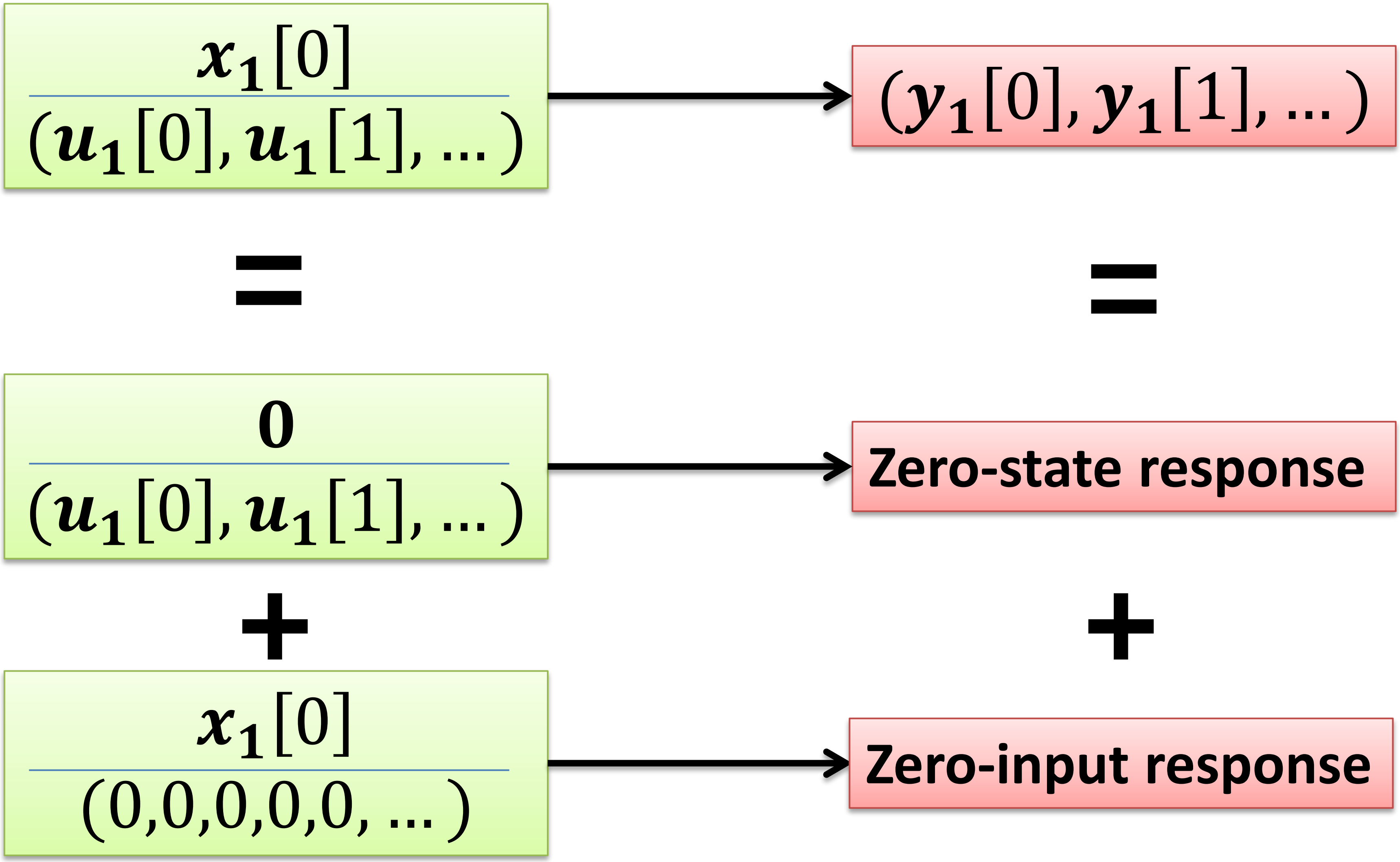
$$\mathcal{F}((\boldsymbol{x}_1, \boldsymbol{u}_1) + \alpha(\boldsymbol{x}_2, \boldsymbol{u}_2)) = \mathcal{F}(\boldsymbol{x}_1, \boldsymbol{u}_1) + \alpha\mathcal{F}(\boldsymbol{x}_2, \boldsymbol{u}_2)$$

$\boldsymbol{x}_1[0] + \boldsymbol{x}_2[0]$

$(\boldsymbol{u}_1[0] + \boldsymbol{u}_2[0],$
$\boldsymbol{u}_1[1] + \boldsymbol{u}_2[1],$
$\boldsymbol{u}_1[2] + \boldsymbol{u}_2[2],$
$\ldots)$

$(\boldsymbol{y}_1[0] + \boldsymbol{y}_2[0],$
$\boldsymbol{y}_1[1] + \boldsymbol{y}_2[1],$
$\boldsymbol{y}_1[2] + \boldsymbol{y}_2[2],$
$\ldots)$


$\frac{\boldsymbol{x}_1[0]}{(\boldsymbol{u}_1[0], \boldsymbol{u}_1[1], \ldots)}$
$(\boldsymbol{y}_1[0], \boldsymbol{y}_1[1], \ldots)$



$\frac{x_1[0]}{(u_1[0], u_1[1], \ldots)}$
$(y_1[0], y_1[1], \ldots)$
=
=
$\frac{0}{(u_1[0], u_1[1], \ldots)}$
Zero-state response
+
+
$\frac{x_1[0]}{(0,0,0,0,0, \ldots)}$
Zero-input response

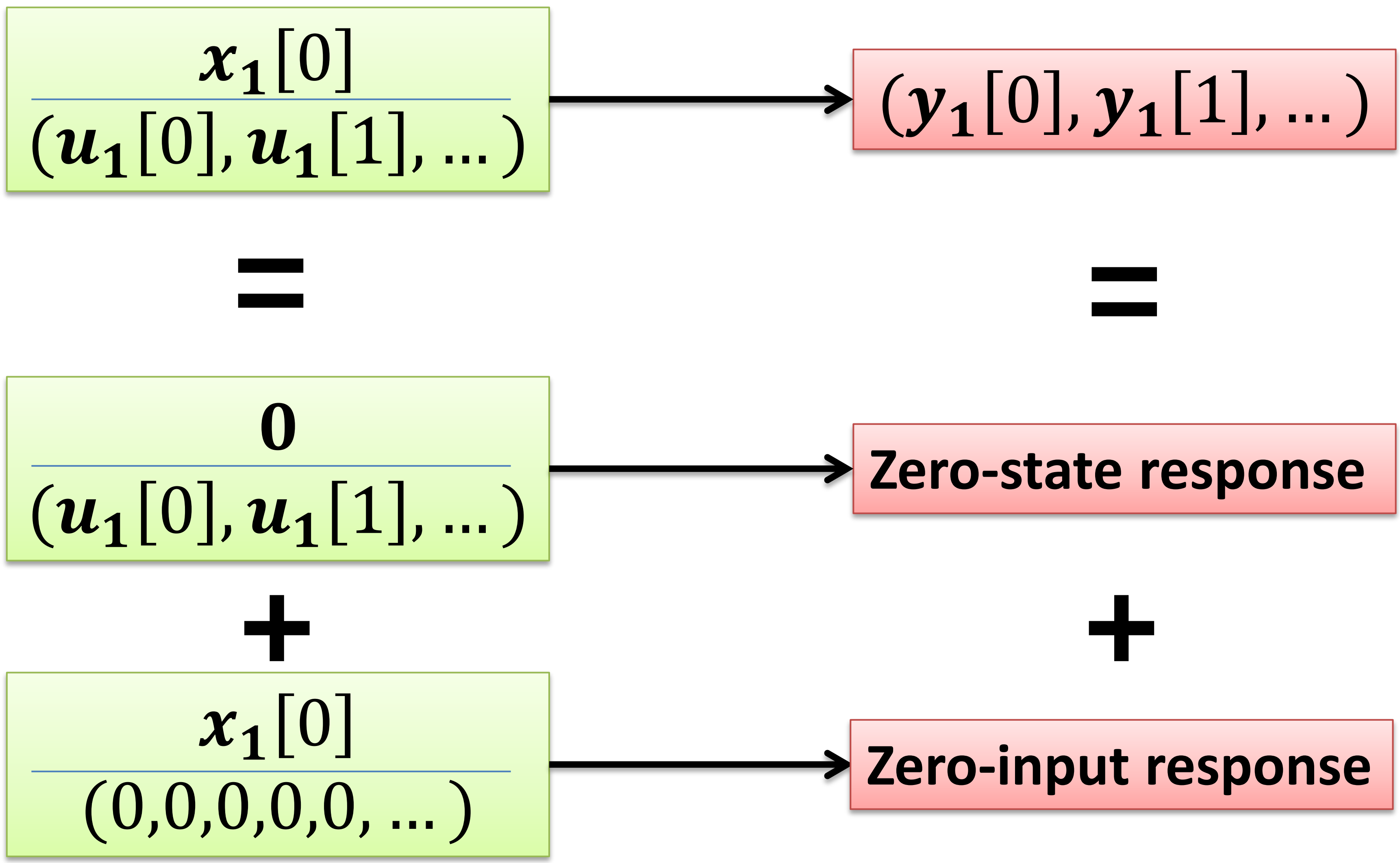
$x_1[0]$
$(u_1[0], u_1[1], \ldots)$ → $(y_1[0], y_1[1], \ldots)$

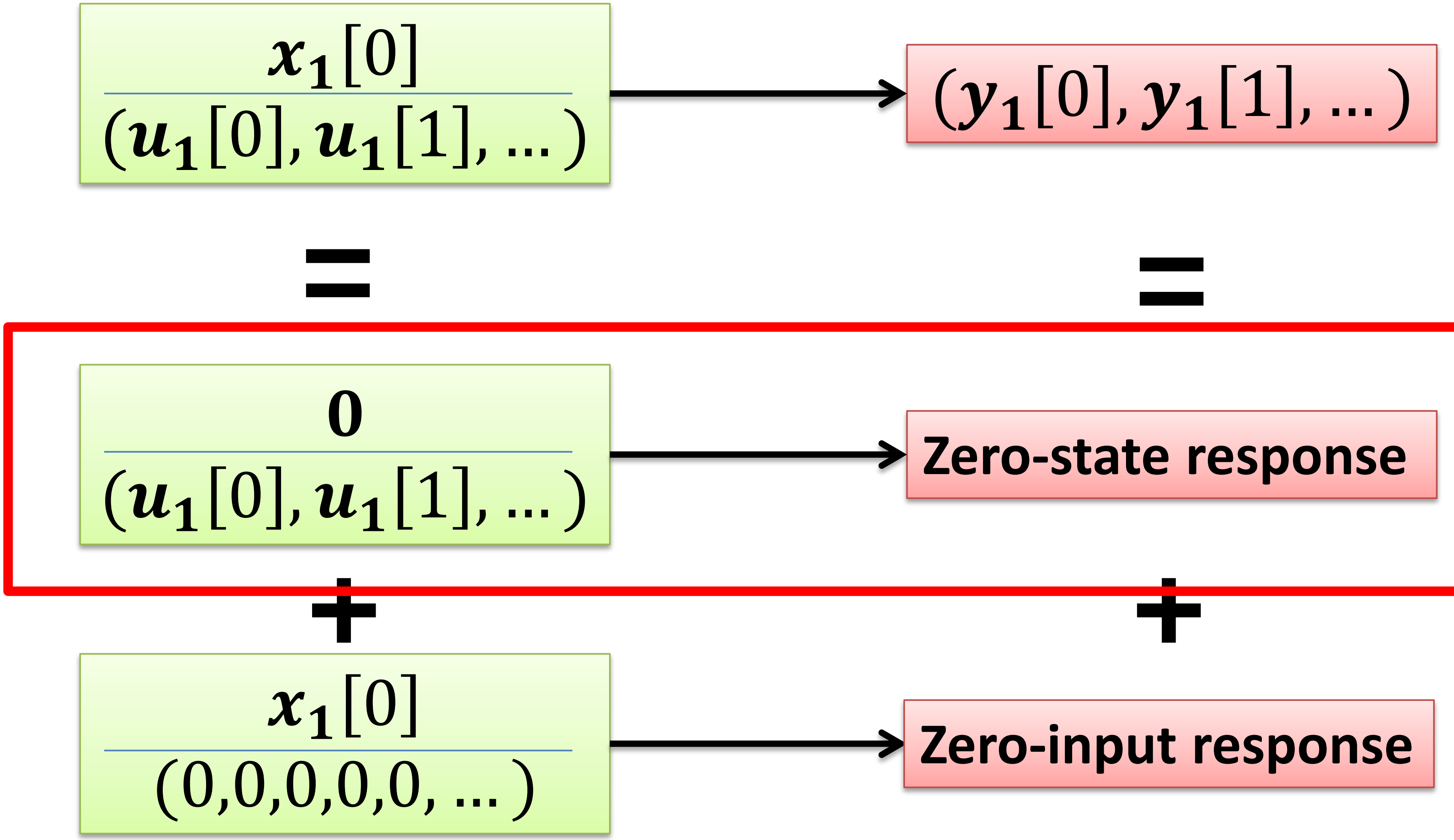
=   =

$$y[t] = y_{zs}[t] + y_{zi}[t]$$

+   +

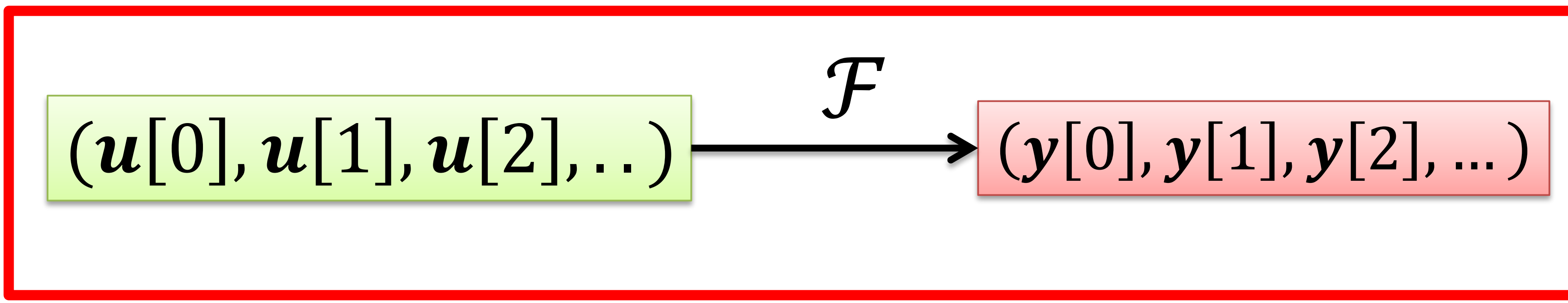
$x_1[0]$
$(0,0,0,0,0, \ldots)$ → Zero-input response

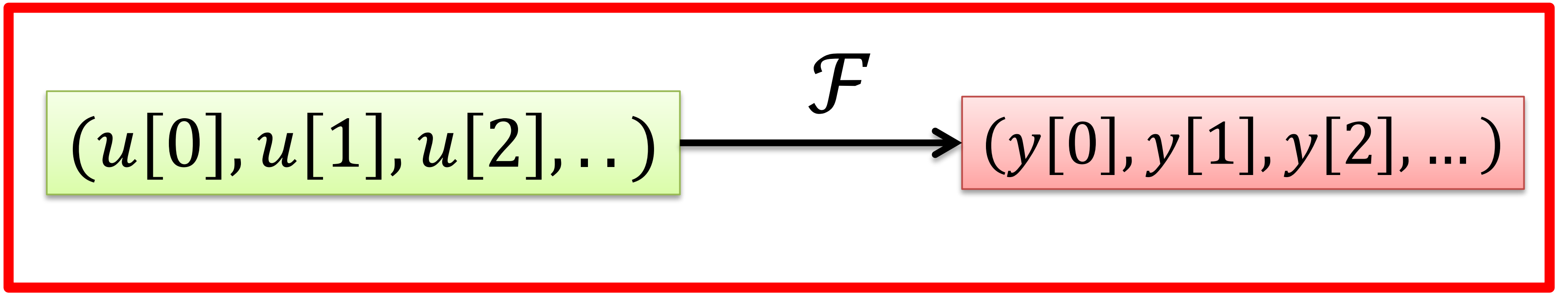

$\frac{x_1[0]}{(u_1[0], u_1[1], \dots)}$
$(y_1[0], y_1[1], \dots)$
=
=
$\frac{0}{(u_1[0], u_1[1], \dots)}$
Zero-state response
+
+
$\frac{x_1[0]}{(0,0,0,0,0, \dots)}$
Zero-input response



$\boldsymbol{x}_1[0]$
$(\boldsymbol{u}_1[0], \boldsymbol{u}_1[1], \dots)$
$(\boldsymbol{y}_1[0], \boldsymbol{y}_1[1], \dots)$
=
=
$\mathbf{0}$
$(\boldsymbol{u}_1[0], \boldsymbol{u}_1[1], \dots)$
Zero-state response
+
+
$\boldsymbol{x}_1[0]$
$(0,0,0,0,0, \dots)$
Zero-input response

$$\frac{\mathbf{0}}{(\boldsymbol{u_1}[0], \boldsymbol{u_1}[1], \dots)}$$
Zero-state response


$(\boldsymbol{u}[0], \boldsymbol{u}[1], \boldsymbol{u}[2], \ldots)$
$\mathcal{F}$
$(\boldsymbol{y}[0], \boldsymbol{y}[1], \boldsymbol{y}[2], \ldots)$


$(u[0], u[1], u[2], \ldots)$
$\mathcal{F}$
$(y[0], y[1], y[2], \ldots)$

$$(u[0], u[1], u[2], \ldots) \xrightarrow{\mathcal{F}} (y[0], y[1], y[2], \ldots)$$

?

$$(u[0], u[1], u[2], \ldots) \xrightarrow{\mathcal{F}} (y[0], y[1], y[2], \ldots)$$

$$=$$

$$(u[0], 0,0,0,0,0, \ldots)$$

$$+$$

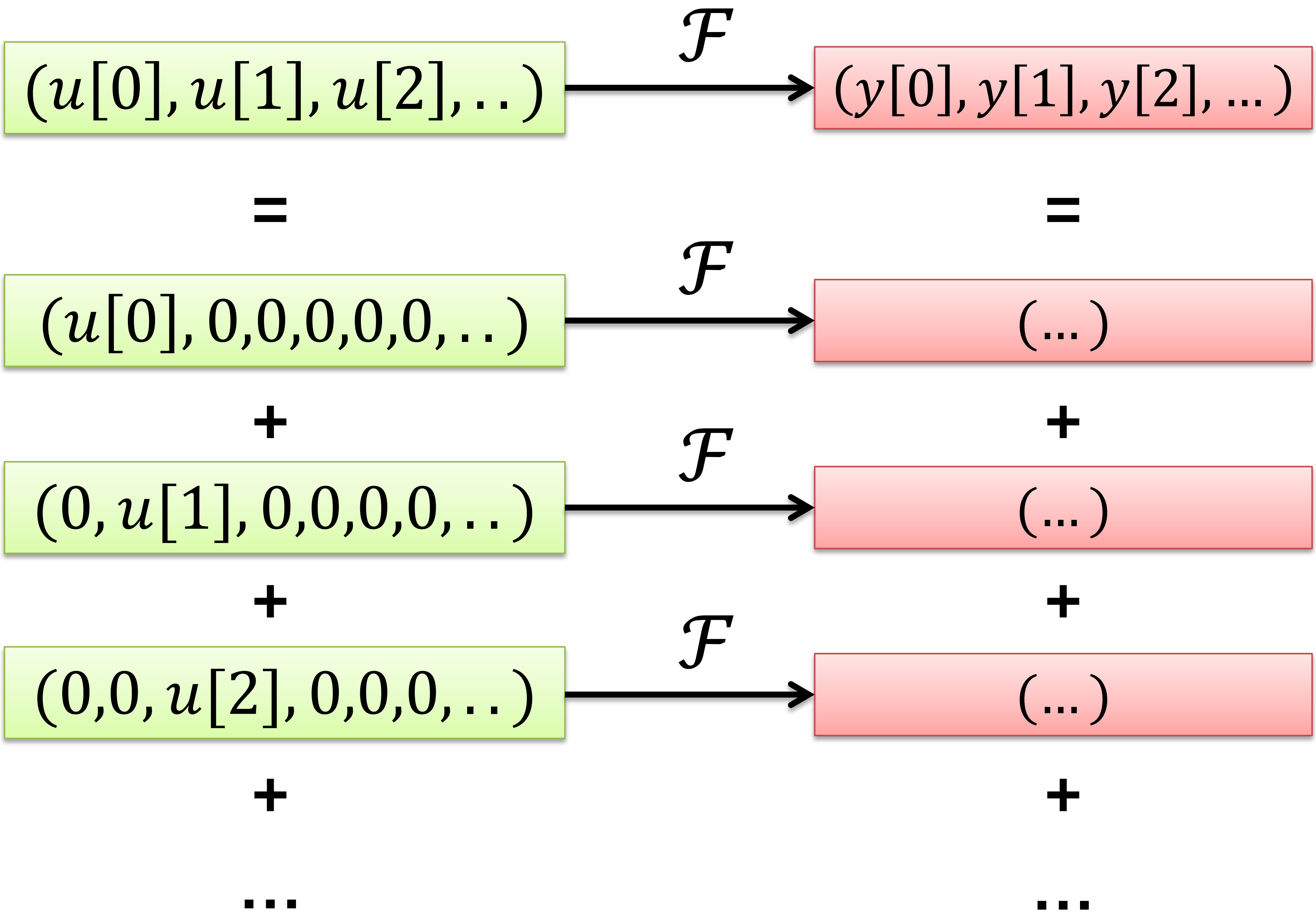
$$(0, u[1], 0,0,0,0, \ldots)$$

$$+$$

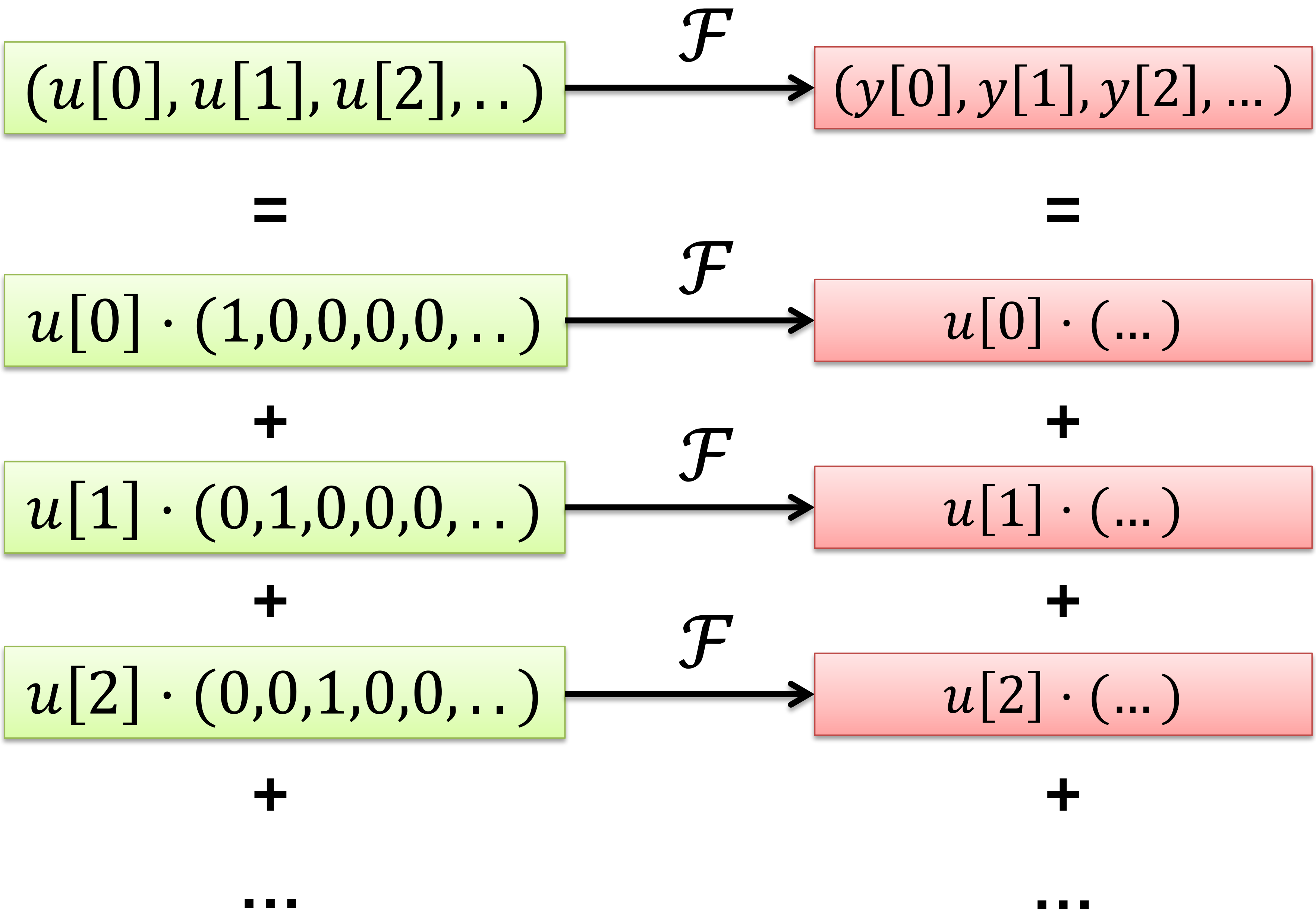
$$(0,0, u[2], 0,0,0, \ldots)$$

$$+$$

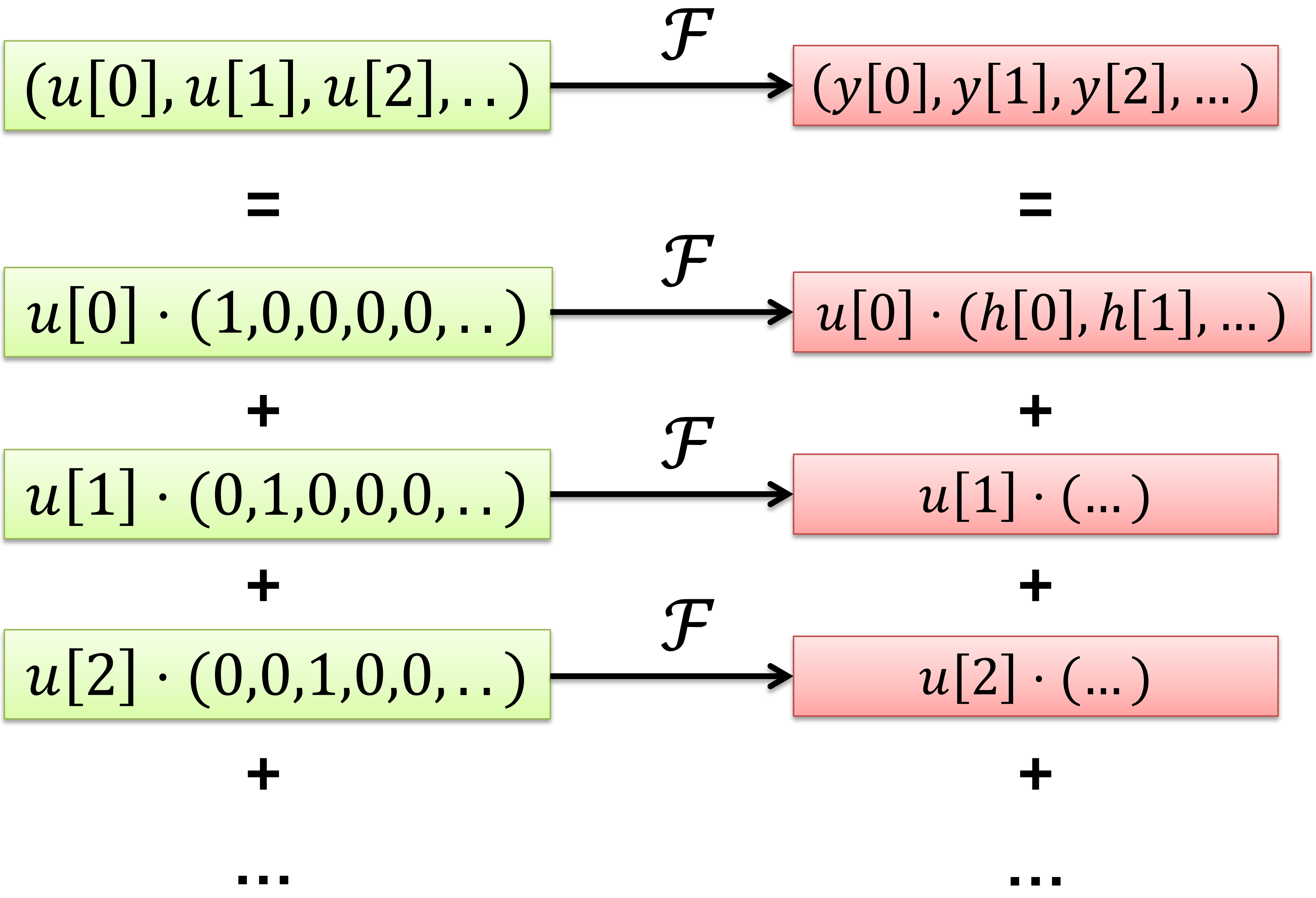
$$\cdots$$



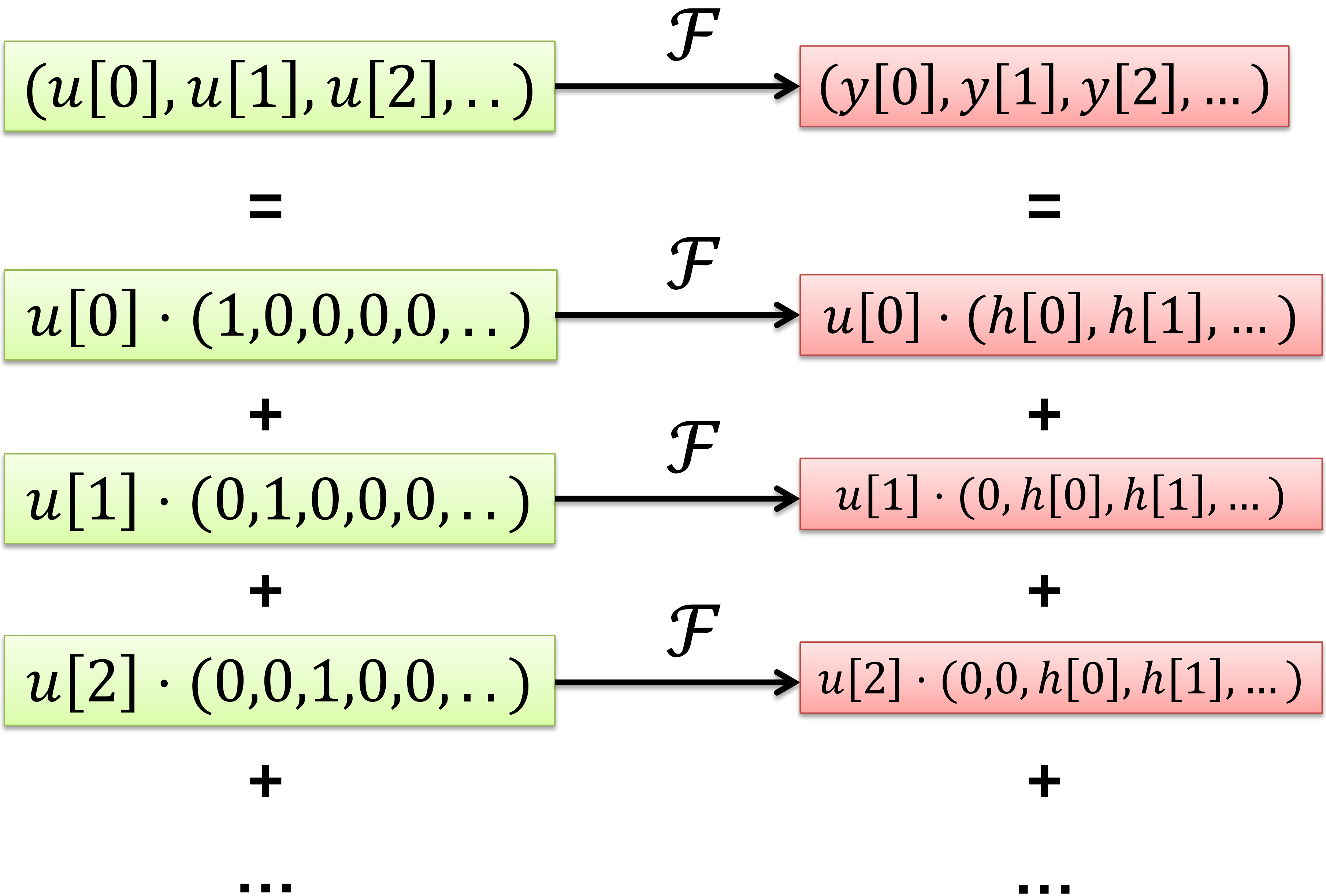
$\mathcal{F}$
$(u[0], u[1], u[2], \ldots)$
$(y[0], y[1], y[2], \ldots)$
=
=
$(u[0], 0,0,0,0,0, \ldots)$
$(\ldots)$
+
+
$(0, u[1], 0,0,0,0, \ldots)$
$(\ldots)$
+
+
$(0,0, u[2], 0,0,0, \ldots)$
$(\ldots)$
+
+
...
...



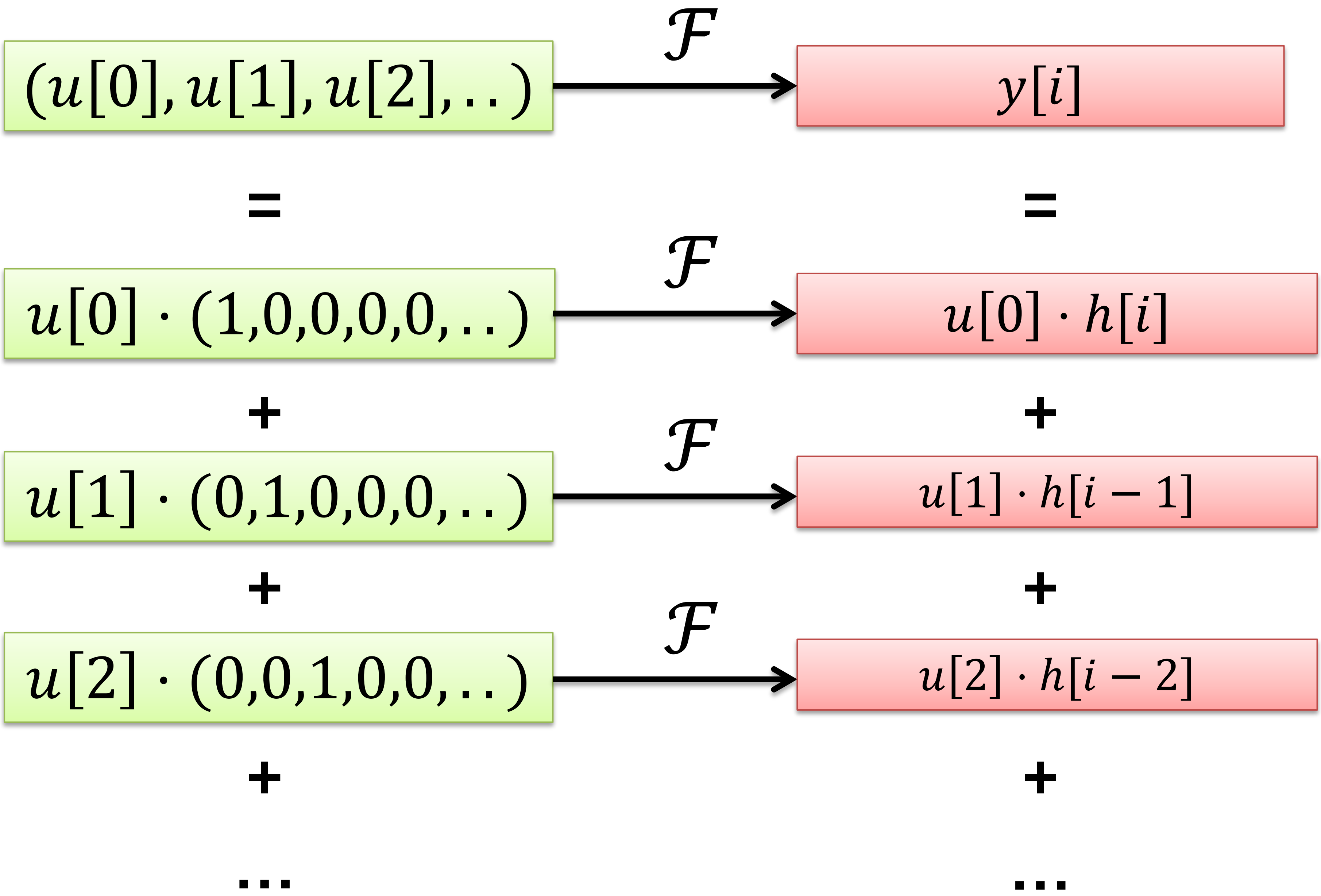
$\mathcal{F}$
$(u[0], u[1], u[2], \ldots)$
$(y[0], y[1], y[2], \ldots)$
=
=
$u[0] \cdot (1,0,0,0,0, \ldots)$
$u[0] \cdot (\ldots)$
+
+
$u[1] \cdot (0,1,0,0,0, \ldots)$
$u[1] \cdot (\ldots)$
+
+
$u[2] \cdot (0,0,1,0,0, \ldots)$
$u[2] \cdot (\ldots)$
+
+
…
…



$(u[0], u[1], u[2], \ldots)$
$\mathcal{F}$
$(y[0], y[1], y[2], \ldots)$
=
=
$u[0] \cdot (1,0,0,0,0, \ldots)$
$\mathcal{F}$
$u[0] \cdot (h[0], h[1], \ldots)$
+
+
$u[1] \cdot (0,1,0,0,0, \ldots)$
$\mathcal{F}$
$u[1] \cdot (\ldots)$
+
+
$u[2] \cdot (0,0,1,0,0, \ldots)$
$\mathcal{F}$
$u[2] \cdot (\ldots)$
+
+
…
…



$(u[0], u[1], u[2], \ldots)$
$\mathcal{F}$
$(y[0], y[1], y[2], \ldots)$
=
=
$u[0] \cdot (1,0,0,0,0, \ldots)$
$\mathcal{F}$
$u[0] \cdot (h[0], h[1], \ldots)$
+
+
$u[1] \cdot (0,1,0,0,0, \ldots)$
$\mathcal{F}$
$u[1] \cdot (0, h[0], h[1], \ldots)$
+
+
$u[2] \cdot (0,0,1,0,0, \ldots)$
$\mathcal{F}$
$u[2] \cdot (0,0, h[0], h[1], \ldots)$
+
+
…
…



$(u[0], u[1], u[2], \ldots)$
$\mathcal{F}$
$y[i]$
=
=
$u[0] \cdot (1,0,0,0,0,\ldots)$
$\mathcal{F}$
$u[0] \cdot h[i]$
+
+
$u[1] \cdot (0,1,0,0,0,\ldots)$
$\mathcal{F}$
$u[1] \cdot h[i-1]$
+
+
$u[2] \cdot (0,0,1,0,0,\ldots)$
$\mathcal{F}$
$u[2] \cdot h[i-2]$
+
+
…
…

$$(u[0], u[1], u[2], \ldots) \xrightarrow{\mathcal{F}} y[i]$$

$$y[i] = u[0]h[i] + u[1]h[i-1] + u[2]h[i-2] + \cdots$$

+ +

$$u[1] \cdot (0,1,0,0,0,\ldots) \xrightarrow{\mathcal{F}} u[1] \cdot h[i-1]$$

+ +

$$u[2] \cdot (0,0,1,0,0,\ldots) \xrightarrow{\mathcal{F}} u[2] \cdot h[i-2]$$

+ +

… …

$(u[0], u[1], u[2], \ldots) \xrightarrow{\mathcal{F}} y[i]$

$$y[i] = \sum_k u[k]h[i-k]$$

$+ \qquad\qquad +$

$u[1] \cdot (0,1,0,0,0,\ldots) \xrightarrow{\mathcal{F}} u[1] \cdot h[i-1]$

$+ \qquad\qquad +$

$u[2] \cdot (0,0,1,0,0,\ldots) \xrightarrow{\mathcal{F}} u[2] \cdot h[i-2]$

$+ \qquad\qquad +$

$\cdots \qquad\qquad \cdots$

$$(u[0], u[1], u[2], \ldots) \xrightarrow{\mathcal{F}} y[i]$$

$$y[i] = \sum_k u[k]h[i-k] =: \text{convolve}(u, h)$$

$+ \qquad\qquad +$

$$u[1] \cdot (0,1,0,0,0,\ldots) \xrightarrow{\mathcal{F}} u[1] \cdot h[i-1]$$

$+ \qquad\qquad +$

$$u[2] \cdot (0,0,1,0,0,\ldots) \xrightarrow{\mathcal{F}} u[2] \cdot h[i-2]$$

$+ \qquad\qquad +$

$\cdots \qquad\qquad \cdots$

$(u[0], u[1], u[2], \ldots) \xrightarrow{\mathcal{F}} y[i]$

$$y = u * h$$

$+ \qquad\qquad +$

$u[1] \cdot (0,1,0,0,0,\ldots) \xrightarrow{\mathcal{F}} u[1] \cdot h[i-1]$

$+ \qquad\qquad +$

$u[2] \cdot (0,0,1,0,0,\ldots) \xrightarrow{\mathcal{F}} u[2] \cdot h[i-2]$

$+ \qquad\qquad +$

$\cdots \qquad\qquad \cdots$

$(u[0], u[1], u[2], \ldots) \xrightarrow{\mathcal{F}} y[i]$

$$y = h * u$$

$+ \qquad\qquad +$

$u[1] \cdot (0,1,0,0,0,\ldots) \xrightarrow{\mathcal{F}} u[1] \cdot h[i-1]$

$+ \qquad\qquad +$

$u[2] \cdot (0,0,1,0,0,\ldots) \xrightarrow{\mathcal{F}} u[2] \cdot h[i-2]$

$+ \qquad\qquad +$

$\cdots \qquad\qquad \cdots$

$(u[0], u[1], u[2], \dots)$ $\xrightarrow{\mathcal{F}}$ $y[i]$

$$y = h * u$$

$$u = (2,3,0,0,0,\dots)$$
$$h = (1,0,1,0,0,\dots)$$
$$h * u = ?$$

…

…

$(u[0], u[1], u[2], \ldots)$ $\xrightarrow{\mathcal{F}}$ $y[i]$

$$y = h * u$$

$$u = (2,3,0,0,0, \ldots)$$
$$h = (1,0,1,0,0, \ldots)$$
$$h * u = (2,3,2,3,0,0,0, \ldots)$$

$\ldots$ $\ldots$

$(u[0], u[1], u[2], \ldots) \xrightarrow{\mathcal{F}} y[i]$

$$y = h * u$$

$+ \qquad +$

$u[1] \cdot (0,1,0 \ldots \qquad \ldots[i-1]$

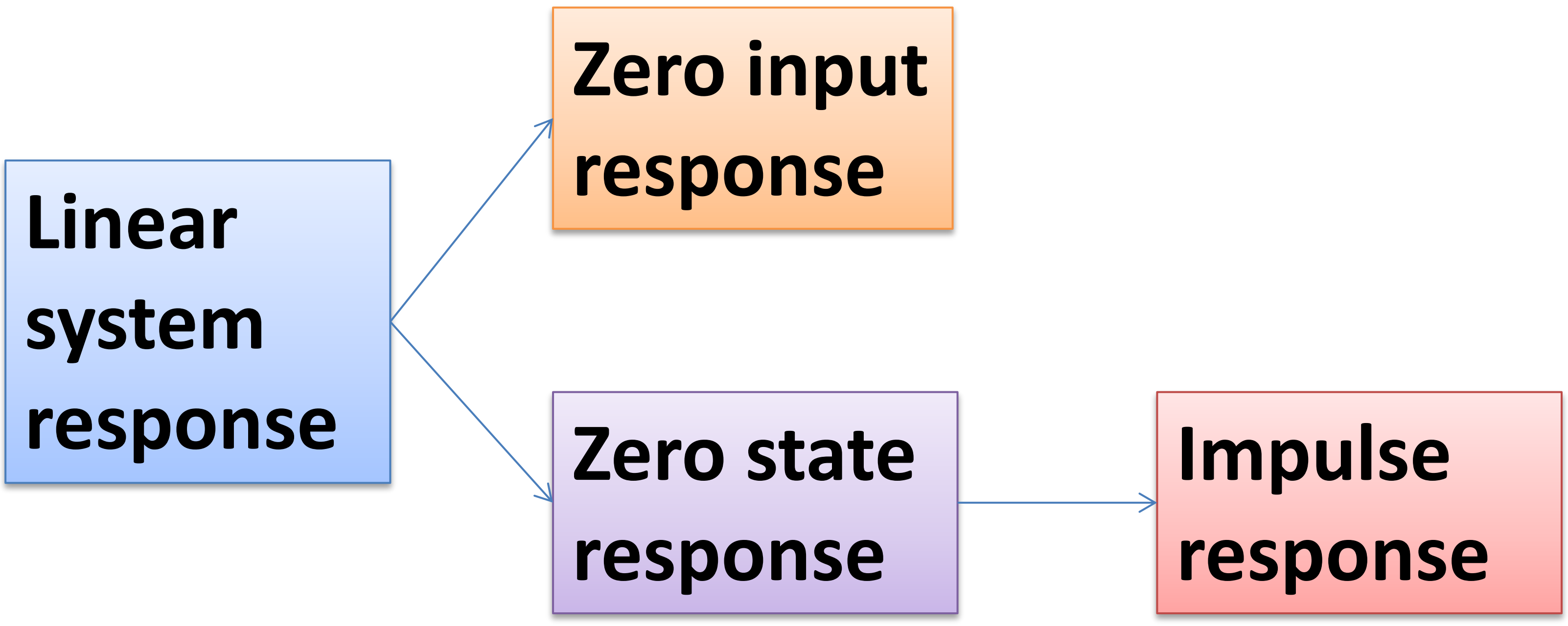
Impulse response

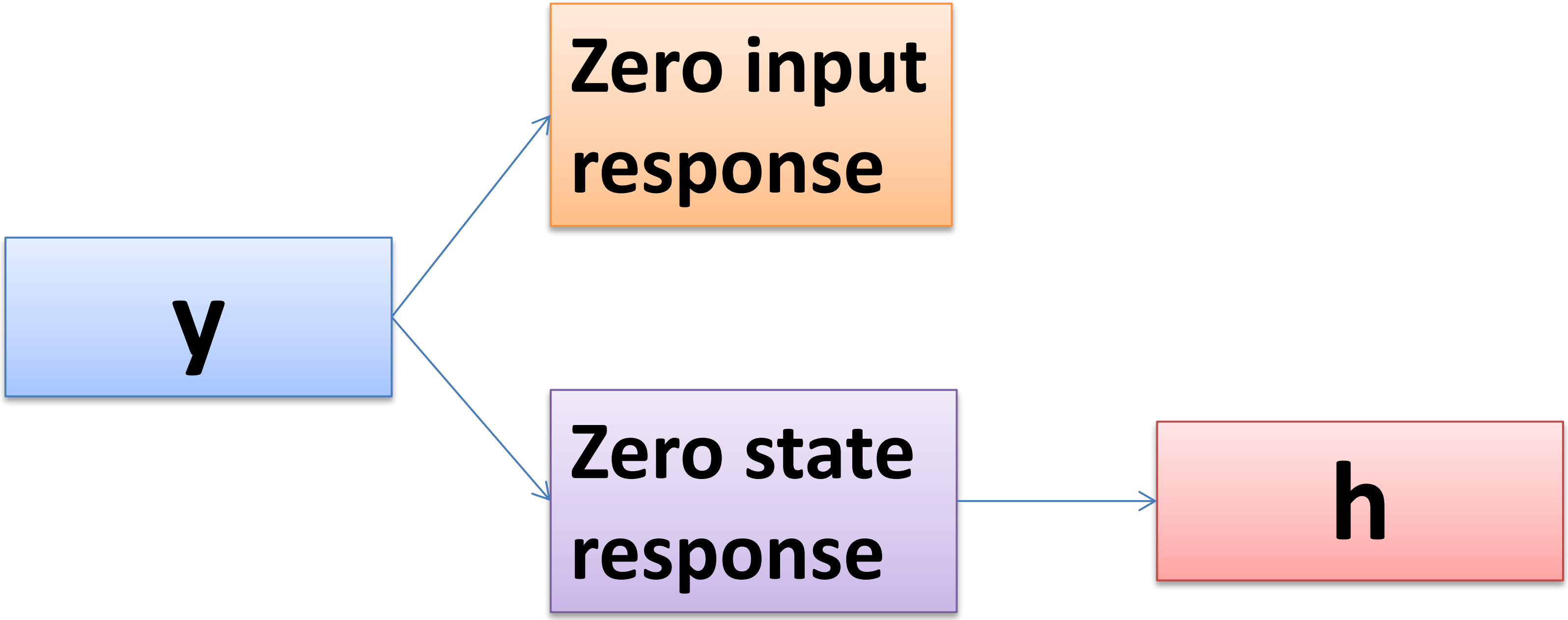
$+$

$u[2] \cdot (0,0,1,0,0,\ldots) \xrightarrow{\mathcal{F}} u[2] \cdot h[i-2]$

$+ \qquad +$

$\cdots \qquad \cdots$

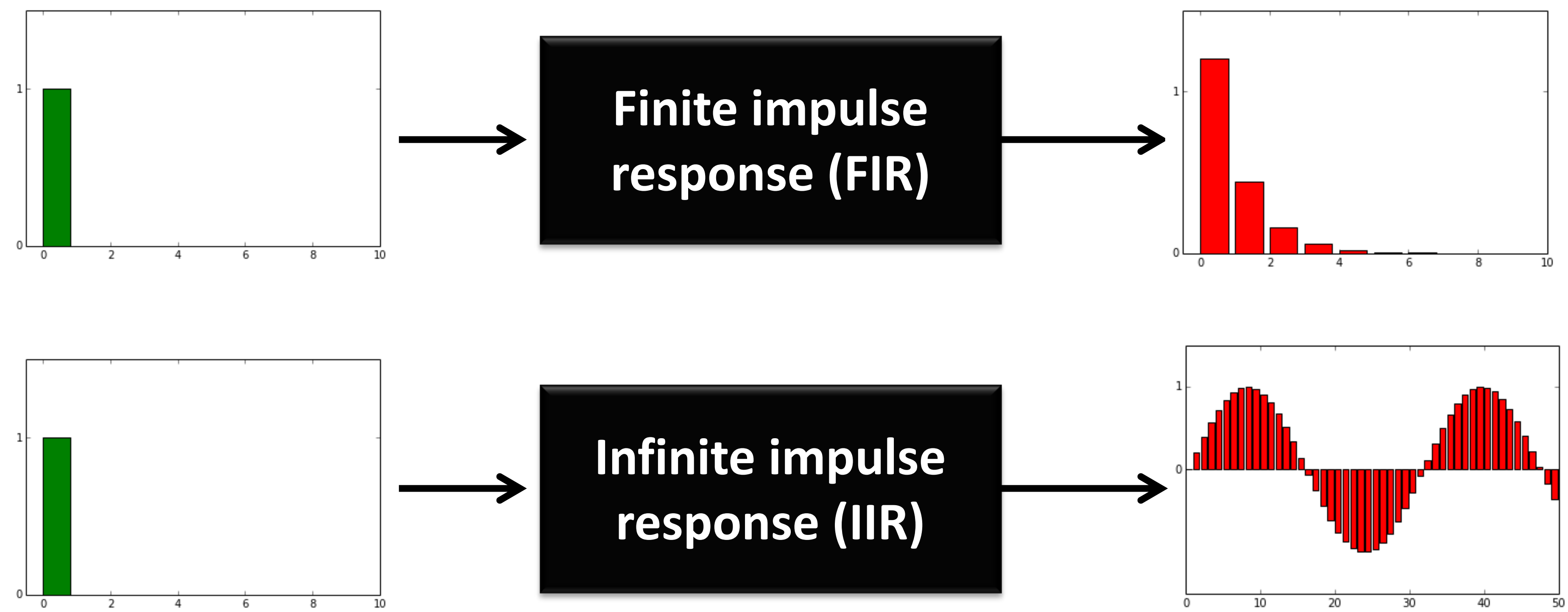

Linear system response
Zero input response
Zero state response
Impulse response

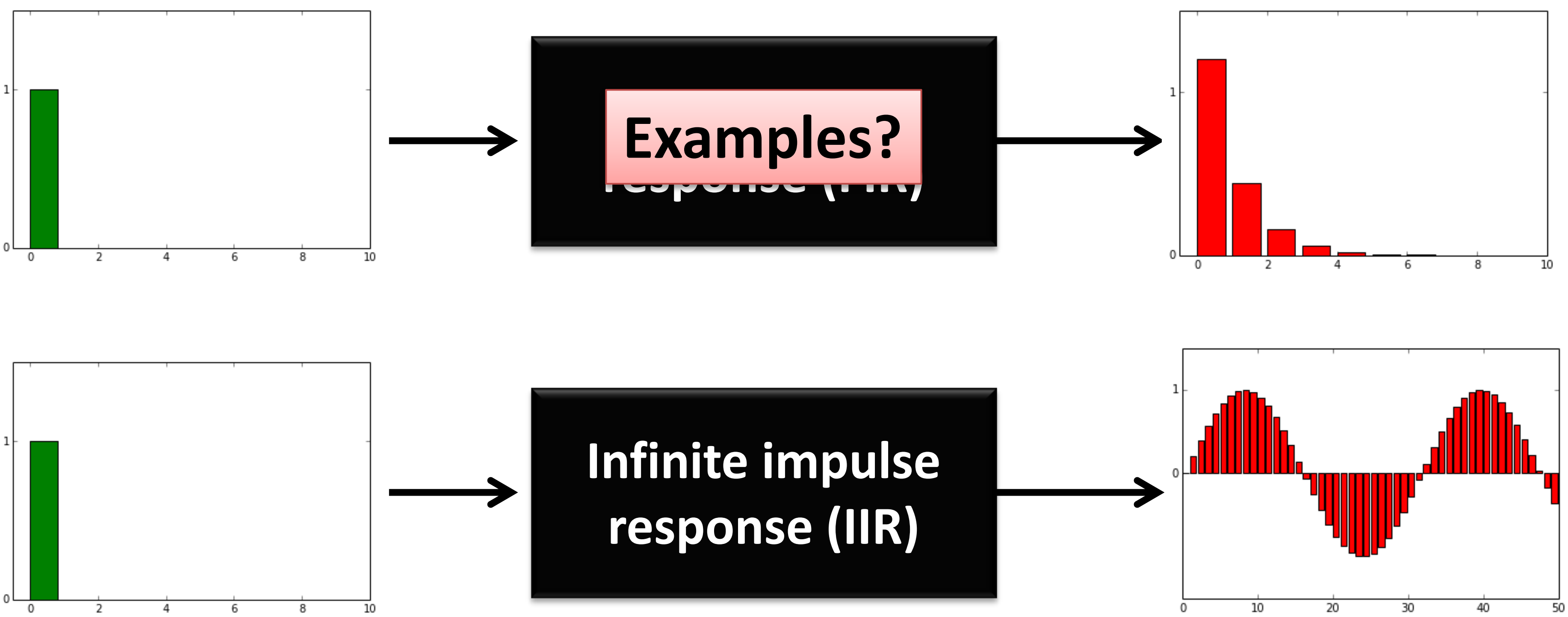

y
Zero input response
Zero state response
h

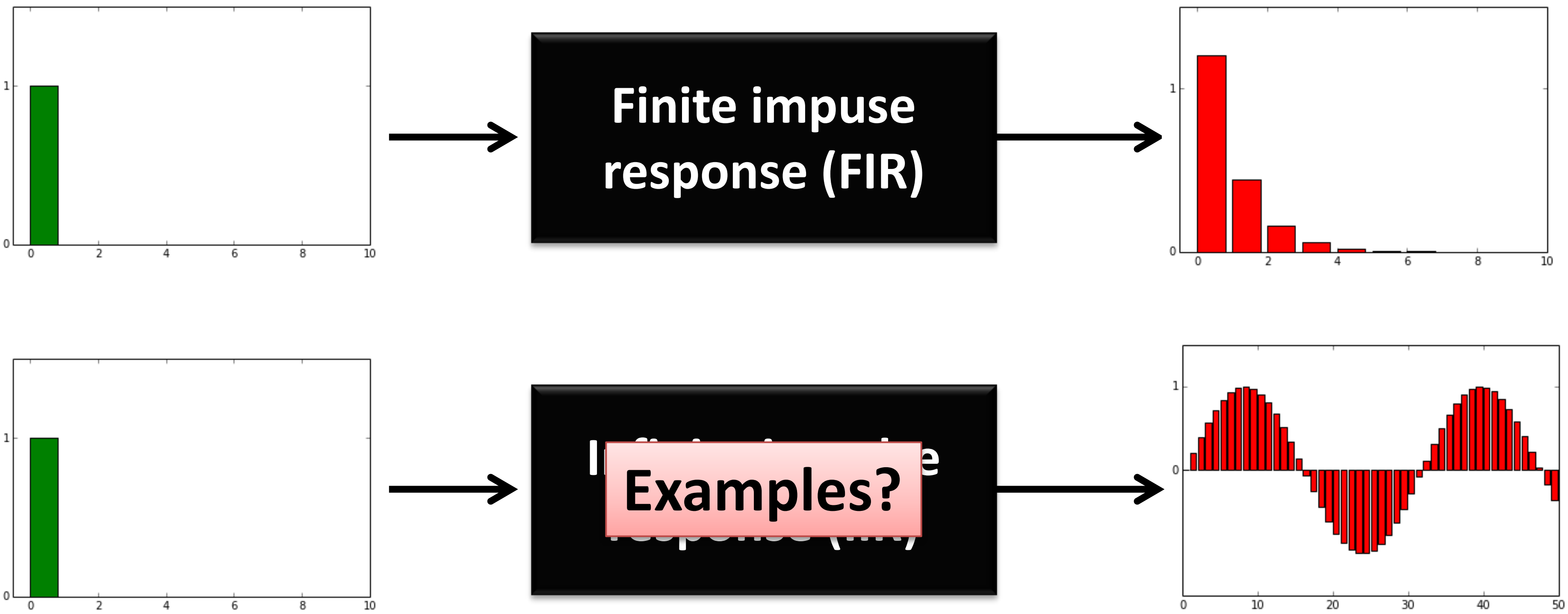
$$y = h * u$$


1
0
0
2
4
6
8
10
1
0
0
2
4
6
8
10

Finite impulse response (FIR)
0
1
0
2
4
6
8
10


Finite impulse response (FIR)
Infinite impulse response (IIR)


Examples?
Infinite impulse
response (IIR)


Finite impuse response (FIR)
Examples?

$$x[t+1] = \boldsymbol{A}x[t] + \boldsymbol{B}u[t]$$
$$y[t] = \boldsymbol{C}x[t] + \boldsymbol{D}u[t]$$

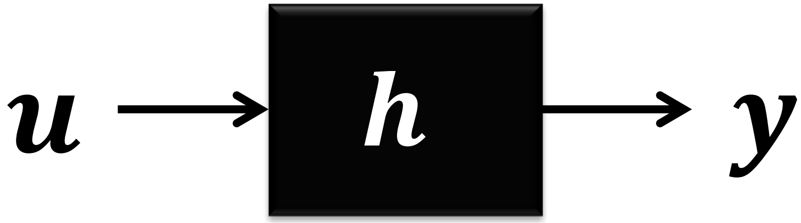
$$\boldsymbol{y} = \boldsymbol{h} * \boldsymbol{u}$$

Discrete | Deterministic | Causal

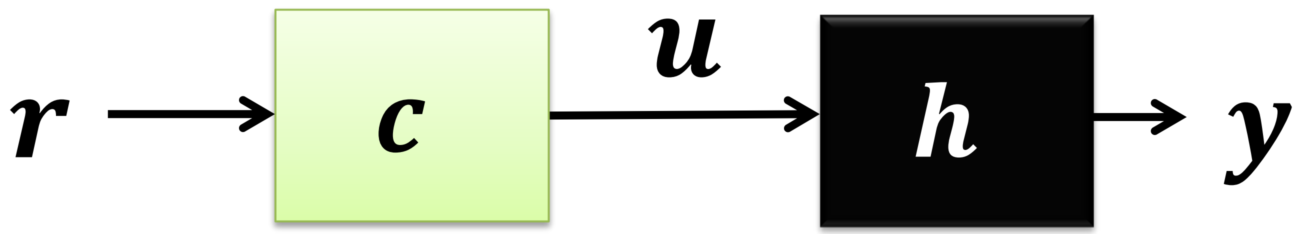
Time-invariant | Finite-dimensional

Linear | Dynamic system

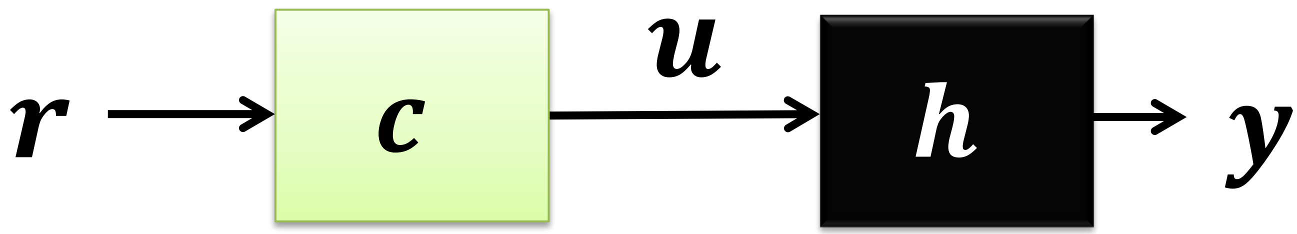
Convolution | Impulse response | FIR/IIR


u
h
y

$= r?$

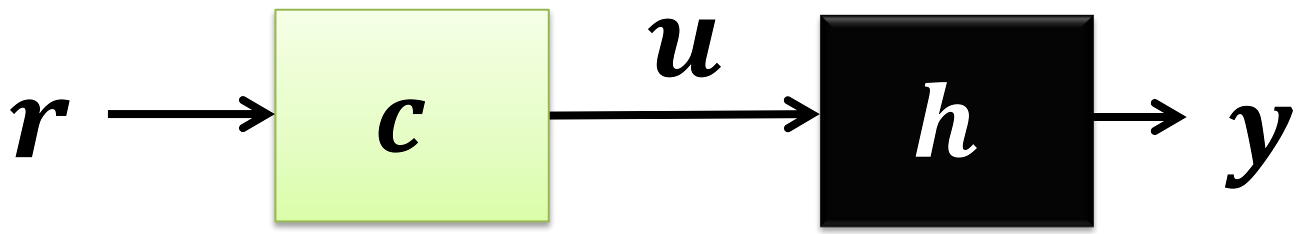


$= r?$



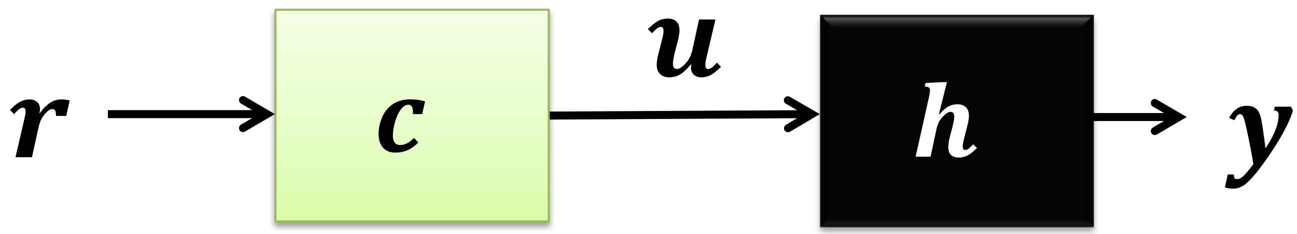
$$= r?$$

$$u = ?$$



$= r?$

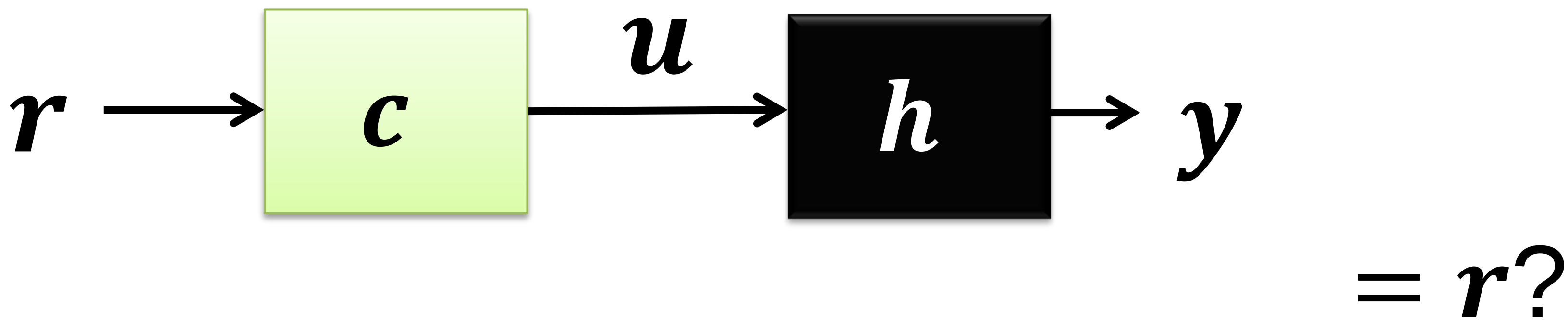
$$u = c * r$$



$= r?$

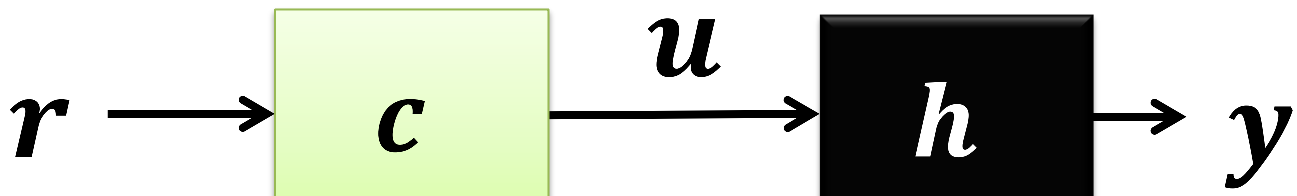
$$u = c * r$$

$$y = h * u$$



$$\boldsymbol{u} = \boldsymbol{c} * \boldsymbol{r}$$

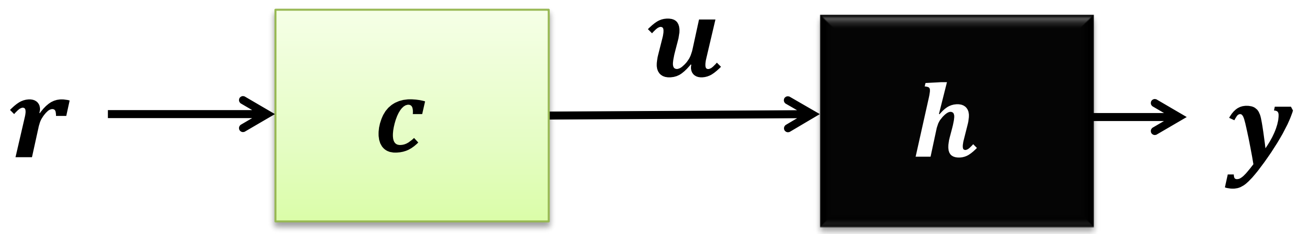
$$\boldsymbol{y} = \boldsymbol{h} * (\boldsymbol{c} * \boldsymbol{r})$$



$= r?$

$$u = c * r$$

$$r \approx h * (c * r)$$

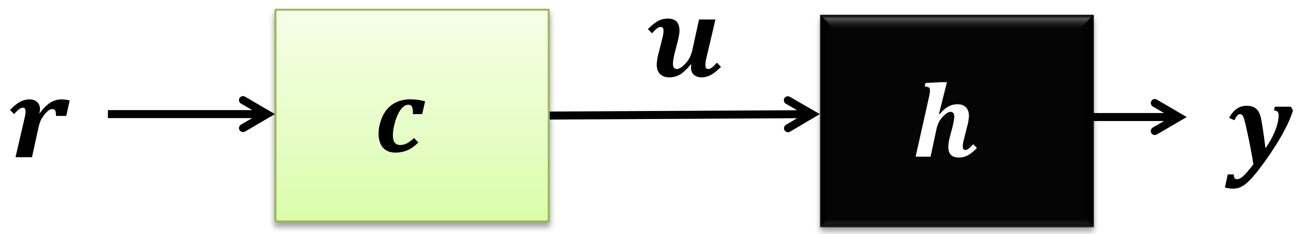


$$= r?$$

$$u = c * r$$

$$r \approx h * (c * r)$$

$$\hat{r} \approx \hat{h} \cdot (\hat{c} \cdot \hat{r})$$



$= r?$

$$u = c * r$$

$$r \approx h * (c * r)$$

$$\hat{r} \approx \widehat{h} \cdot (\hat{c} \cdot \hat{r})$$

$$\hat{c} \approx \widehat{h}^{-1}$$

$$\boldsymbol{y} = \boldsymbol{h} * \boldsymbol{u}$$

$$\Updownarrow$$

$$\widehat{\boldsymbol{y}} = \widehat{\boldsymbol{h}} \cdot \widehat{\boldsymbol{u}}$$

$$y = h * u$$

Fourier transform

Laplace transform

Z-transform

$$\hat{y} = \hat{h} \cdot \hat{u}$$

$$\boldsymbol{h} = (1, 2, 0, 1, 3, -1, \dots)$$

$$\boldsymbol{h} = (1, 2, 0, 1, 3, -1, \ldots)$$

$$\widehat{\boldsymbol{h}}(z) = 1 \cdot z^{0} + 2 \cdot z^{-1} + 0 \cdot z^{-2} + \cdots$$

$$\boldsymbol{u} = (1\,, 0, 1, 0,0, \ldots)$$

$$\boldsymbol{u} = (1\,,0,1,0,0,\ldots)$$

$$\widehat{\boldsymbol{u}}(z) = 1 \cdot z^{0} + 0 \cdot z^{-1} + 1 \cdot z^{-2} + \cdots$$

$$\boldsymbol{u} = (1\,, 0, 1, 0, 0, \ldots)$$

$$\widehat{\boldsymbol{u}}(z) = 1 + \frac{1}{z^2}$$

$$\boldsymbol{u} = (1\,, 0, 1, 0,0, \ldots)$$

Z-transform

$$\widehat{\boldsymbol{u}}(z) = 1 + \frac{1}{z^2}$$

$$\boldsymbol{u} = (1\,,0,1,0,0,\ldots)\quad \boldsymbol{h} = (2\,,3,0,0,0,\ldots)$$

$$\widehat{\boldsymbol{u}}(z) = 1 + \frac{1}{z^2} \qquad \widehat{\boldsymbol{h}}(z) = 2 + \frac{3}{z}$$

$$\boldsymbol{u} = (1\,, 0, 1, 0, 0, \ldots) \quad \boldsymbol{h} = (2\,, 3, 0, 0, 0, \ldots)$$

$$\widehat{\boldsymbol{u}}(z) = 1 + \frac{1}{z^2} \qquad \widehat{\boldsymbol{h}}(z) = 2 + \frac{3}{z}$$

$$\widehat{\boldsymbol{u}}(z)\widehat{\boldsymbol{h}}(z) = \left(1 + \frac{1}{z^2}\right)\left(2 + \frac{3}{z}\right)$$

$$\boldsymbol{u} = (1\,, 0, 1, 0,0, \ldots)\ \boldsymbol{h} = (2\,, 3, 0, 0,0, \ldots)$$

$$\widehat{\boldsymbol{u}}(z) = 1 + \frac{1}{z^2} \qquad \widehat{\boldsymbol{h}}(z) = 2 + \frac{3}{z}$$

$$\widehat{\boldsymbol{u}}(z)\widehat{\boldsymbol{h}}(z) = \left(1 + \frac{1}{z^2}\right)\left(2 + \frac{3}{z}\right)$$

$$\widehat{\boldsymbol{u}}(z)\widehat{\boldsymbol{h}}(z) = 2 + \frac{3}{z} + \frac{2}{z^2} + \frac{3}{z^3}$$

$$\boldsymbol{y} = \boldsymbol{h} * \boldsymbol{u}$$

$$\updownarrow$$

$$\widehat{\boldsymbol{y}}(z) = \widehat{\boldsymbol{h}}(z) \cdot \widehat{\boldsymbol{u}}(z)$$

$$\boldsymbol{y} = \boldsymbol{h} * \boldsymbol{u}$$

Transfer function

$$\widehat{\boldsymbol{y}}(z) = \widehat{\boldsymbol{h}}(z) \cdot \widehat{\boldsymbol{u}}(z)$$

Transfer function of a linear system is the *Z-transform* of its ___________

Transfer function of a linear system is the *Z-transform* of its **impulse response**

$$y[i] = u[i]$$

$$y[i] = u[i]$$

$$\boldsymbol{y} = \boldsymbol{u}$$

$$y[i] = u[i]$$

$$\boldsymbol{y} = \boldsymbol{u}$$

$$\hat{\boldsymbol{y}}(z) = \hat{\boldsymbol{u}}(z)$$

$$y[i] = u[i]$$

$$\boldsymbol{y} = \boldsymbol{u}$$

$$\hat{\boldsymbol{y}}(z) = \hat{\boldsymbol{u}}(z)$$

$$\hat{\boldsymbol{h}}(z)\hat{\boldsymbol{u}}(z) = \hat{\boldsymbol{u}}(z)$$

$$y[i] = u[i]$$

$$\boldsymbol{y} = \boldsymbol{u}$$

$$\widehat{\boldsymbol{y}}(z) = \widehat{\boldsymbol{u}}(z)$$

$$\widehat{\boldsymbol{h}}(z)\widehat{\boldsymbol{u}}(z) = \widehat{\boldsymbol{u}}(z)$$

$$\widehat{\boldsymbol{h}}(z) = 1$$

$$y[i] = u[i - 1]$$

$$y[i] = u[i-1]$$

$$u = (1,2,3,4,5,\dots)$$

$$y[i] = u[i-1]$$

$$u = (1,2,3,4,5,\ldots)$$
$$\hat{u}(z) = 1 + 2z^{-1} + 3z^{-2} + \cdots$$

$$y[i] = u[i-1]$$

$$u = (1,2,3,4,5,\ldots)$$
$$\hat{u}(z) = 1 + 2z^{-1} + 3z^{-2} + \cdots$$

$$y = (0,1,2,3,4,\ldots)$$

$$y[i] = u[i-1]$$

$$u = (1,2,3,4,5,\ldots)$$

$$\hat{u}(z) = 1 + 2z^{-1} + 3z^{-2} + \cdots$$

$$y = (0,1,2,3,4,\ldots)$$

$$\hat{y}(z) = 0 + 1z^{-1} + 2z^{-2} + \cdots$$

$$y[i] = u[i-1]$$

$$u = (1,2,3,4,5,\ldots)$$
$$\hat{u}(z) = 1 + 2z^{-1} + 3z^{-2} + \cdots$$

$$y = (0,1,2,3,4,\ldots)$$
$$\hat{y}(z) = 0 + 1z^{-1} + 2z^{-2} + \cdots$$

$$\hat{y}(z) = \hat{h}(z) \cdot \hat{u}(z)$$

$$y[i] = u[i-1]$$

$$u = (1,2,3,4,5,\ldots)$$

$$\hat{u}(z) = 1 + 2z^{-1} + 3z^{-2} + \cdots$$

$$y = (0,1,2,3,4,\ldots)$$

$$\hat{y}(z) = 0 + 1z^{-1} + 2z^{-2} + \cdots$$

$$\hat{y}(z) = z^{-1} \cdot \hat{u}(z)$$

$$y[i] = u[i-1]$$

$$\hat{h}(z) = z^{-1}$$

$$y[i] = 2u[i-1]$$

$$\hat{h}(z) = 2z^{-1}$$

$$y[i] = 2u[i-3]$$

$$\hat{h}(z) = 2z^{-3}$$

$$y[i] = u[i] + u[i-1] + 2u[i-3]$$

$$\hat{h}(z) = 1 + z^{-1} + 2z^{-3}$$

$$y[i] = u[i] + y[i - 1]$$

$$y[i] = u[i] + y[i-1]$$

$$\boldsymbol{y} = \boldsymbol{u} + \boldsymbol{y}^{(-\mathbf{1})}$$

$$\widehat{\boldsymbol{y}}(z) = \widehat{\boldsymbol{u}}(z) + z^{-1}\widehat{\boldsymbol{y}}(z)$$

$$\widehat{\boldsymbol{y}}(z)(1 - z^{-1}) = \widehat{\boldsymbol{u}}(z)$$

$$\widehat{\boldsymbol{h}}(z)\widehat{\boldsymbol{u}}(z)(1 - z^{-1}) = \widehat{\boldsymbol{u}}(z)$$

$$y[i] = u[i] + y[i-1]$$

$$\hat{h}(z) = \frac{1}{1 - z^{-1}}$$

$$y[i] = u[i] + y[i-1] + 2y[i-2]$$

$$\hat{h}(z) = \frac{1}{1-(z^{-1}+2z^{-2})}$$

$$y[i] = 2u[i] - 3u[i-1]$$
$$+4y[i-1] - 5\mathrm{y}[\mathrm{i} - 2]$$

$$y[i] = 2u[i] - 3u[i-1] \\ \qquad +4y[i-1] - 5\mathrm{y}[\mathrm{i}-2]$$

$$\hat{h}(z) = \frac{2 - 3z^{-1}}{1 - (4z^{-1} - 5z^{-2})}$$

$$y[i] = 2u[i] - 3u[i-1] + 4y[i-1] - 5\mathrm{y}[\mathrm{i}-2]$$

$$\hat{h}(z) = \frac{2 - 3z^{-1}}{1 - (4z^{-1} - 5z^{-2})}$$

$$y[i] = 2u[i] - 3u[i-1] + 4y[i-1] - 5\mathrm{y}[\mathrm{i}-2]$$

$$\hat{h}(z) = \frac{2 - 3z^{-1}}{1 - (4z^{-1} - 5z^{-2})}$$

$$y[i] = 2u[i] - 3u[i-1] + 4y[i-1] - 5\mathrm{y}[\mathrm{i}-2]$$

$$\hat{h}(z) = \frac{2 - 3z^{-1}}{1 - (4z^{-1} - 5z^{-2})}$$

Causality

$$y[i] = 2u[i] - 3u[i-1] + 4y[i-1] - 5y[i-2]$$

$$\hat{h}(z) = \frac{2 - 3z^{-1}}{1 - (4z^{-1} - 5z^{-2})}$$

Infinite impulse response

$$y[i] = 2u[i] - 3u[i-1]$$
$$+4y[i-1] - 5\mathrm{y}[\mathrm{i}-2]$$

$$\hat{h}(z) = \frac{2 - 3z^{-1}}{1 - (4z^{-1} - 5z^{-2})}$$

$z = 2 \pm i$

BIBO unstable

BIBO-stability

- A dynamic system is **BIBO-stable**, if any **bounded input** sequence excites a **bounded output** sequence.

BIBO-stability

- A dynamic system is **BIBO-stable**, if any **bounded input** sequence excites a **bounded output** sequence.

If the impulse response is **absolutely summable**,

$$\sum_t |h[t]| < \infty$$

the system is BIBO-stable.

BIBO-stability

- A dynamic system is **BIBO-stable**, if any **bounded input** sequence excites a **bounded output** sequence.

If the impulse response is **absolutely summable**,

$$\sum_t |h[t]| < \infty$$

the system is BIBO-stable.

If all poles of the transfer function $\widehat{\boldsymbol{h}}(z)$ lie inside the unit circle, the system is BIBO-stable.

$$x[t+1] = \boldsymbol{A}x[t] + \boldsymbol{B}u[t]$$

$$y[t] = \boldsymbol{C}x[t] + \boldsymbol{D}u[t]$$

$$\boldsymbol{y} = \boldsymbol{h} * \boldsymbol{u}$$

Discrete **Deterministic** **Causal**

Time-invariant **Finite-dimensional**

Linear **Dynamic system**

Convolution **Impulse response** **FIR/IIR**

Z-transform **Transfer function**

System modeling & identification

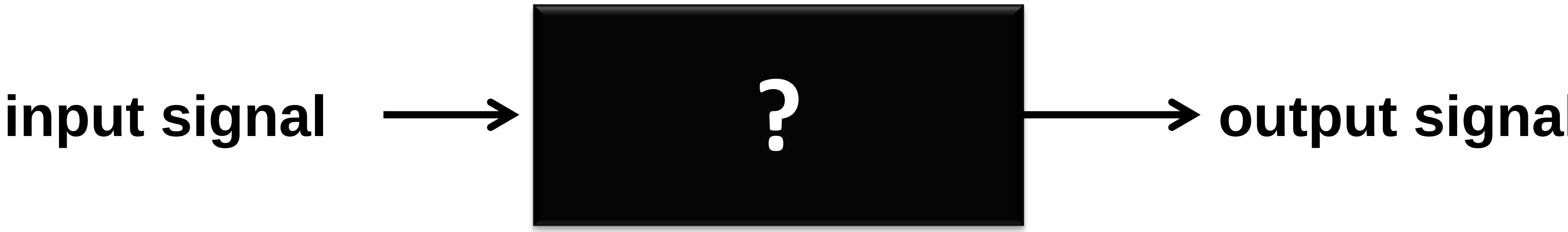
System control

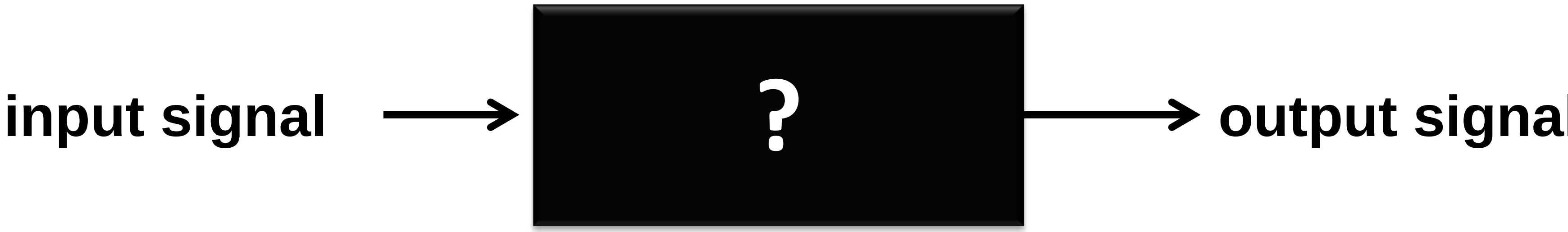
System modeling

- At least four ways to represent linear systems:

System modeling

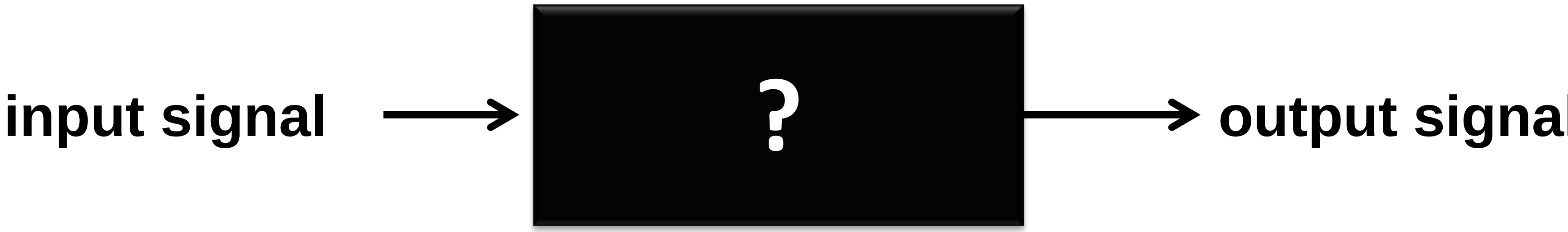
- At least four ways to represent linear systems:
  - State space
    $$x[t+1] = \boldsymbol{A}x[t] + \boldsymbol{B}u[t]$$
    $$y[t] = \boldsymbol{C}x[t] + \boldsymbol{D}u[t]$$
  - Impulse response
    $$\boldsymbol{y} = \boldsymbol{h} * \boldsymbol{u}$$
  - Transfer function
    $$\widehat{\boldsymbol{y}}(z) = \widehat{\boldsymbol{h}}(z)\widehat{\boldsymbol{u}}(z)$$
  - Recursive equation
    $$y[i] = \alpha_0 u[i] + \alpha_1 u[i-1] + \beta_1 y[i-1] + \beta_2 y[i-2] \ldots$$


input signal
?
output signal



$\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ **?** $\boldsymbol{h}[t]$ **?** $\widehat{\boldsymbol{h}}(z)$ **?** $\alpha_0, \alpha_1, \ldots$ **?**

System identification



$\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ **?** $\boldsymbol{h}[t]$**?** $\widehat{\boldsymbol{h}}(z)$ **?** $\alpha_0, \alpha_1, \ldots$ **?**

System identification

- Collect data
  - $u[0], u[1], u[2], u[3], \ldots$
  - $y[0], y[1], y[2], y[3], \ldots$

- Collect data
  - $u[0], u[1], u[2], u[3], \ldots$
  - $y[0], y[1], y[2], y[3], \ldots$
- Select model structure & convert data to
  - $(u[2], u[1], u[0], y[1], y[0]) \rightarrow y[2]$
  - $(u[3], u[2], u[1], y[2], y[1]) \rightarrow y[3]$
  - $(u[4], u[3], u[2], y[3], y[2]) \rightarrow y[4]$
  - …

- Collect data
  - $u[0], u[1], u[2], u[3], \ldots$
  - $y[0], y[1], y[2], y[3], \ldots$
- Select model structure & convert data to
  - $(u[2], u[1], u[0], y[1], y[0]) \rightarrow y[2]$
  - $(u[3], u[2], u[1], y[2], y[1]) \rightarrow y[3]$
  - $(u[4], u[3], u[2], y[3], y[2]) \rightarrow y[4]$
  - …
- Fit linear regression model:

$$y[i] = \alpha_0 u[i] + \alpha_1 u[i-1] + \alpha_2 u[i-2] + \beta_1 y[i-1] + \beta_2 y[i-2]$$

System modeling & identification

System control

Bank deposit

$$y[t] = 1.01y[t-1] + u[t]$$

Bank deposit

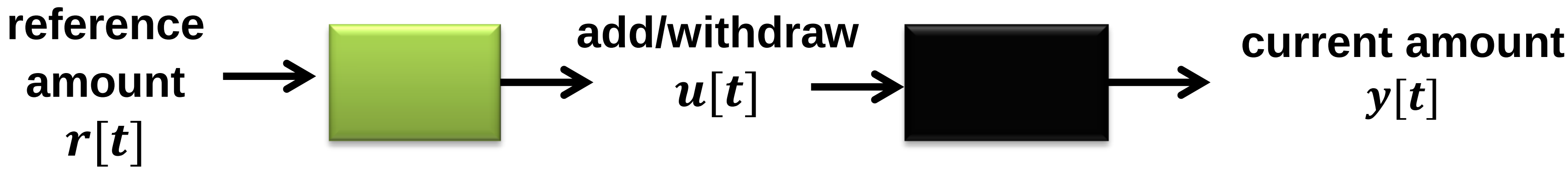
add/withdraw money $u[t]$

current amount $y[t]$

$$y[t] = 1.01y[t-1] + u[t]$$

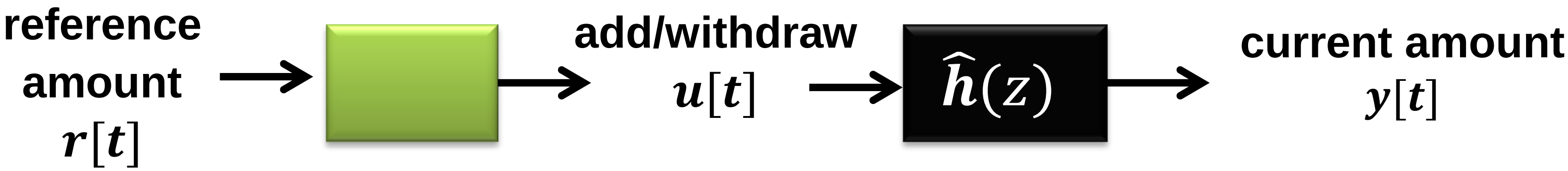
FIR/IIR? **BIBO?**

Bank deposit



$$y[t] = 1.01y[t-1] + u[t]$$

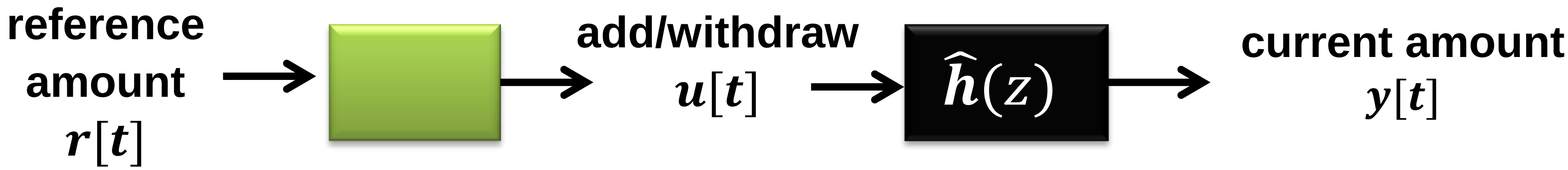
Bank deposit



$$\boldsymbol{y[t] = 1.01y[t-1] + u[t]}$$

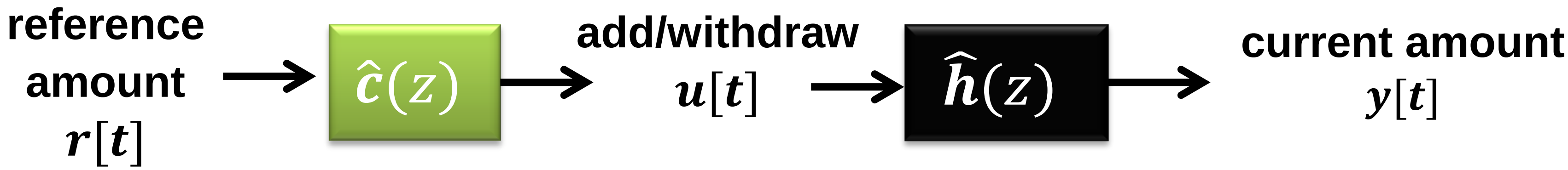
$$\hat{\boldsymbol{h}}(z) = \frac{1}{1 - 1.01z^{-1}}$$

Bank deposit



$$\hat{h}(z) = \frac{1}{1 - 1.01z^{-1}}$$

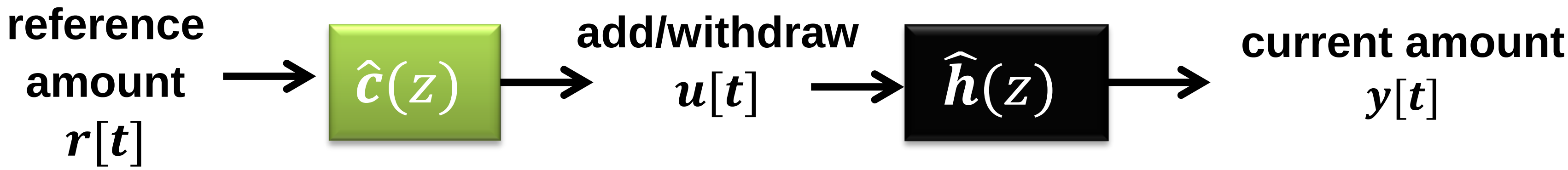
Bank deposit



$$\hat{\boldsymbol{h}}(z) = \frac{1}{1 - 1.01z^{-1}}$$

$$\hat{\boldsymbol{c}}(z)\hat{\boldsymbol{h}}(z) = 1$$

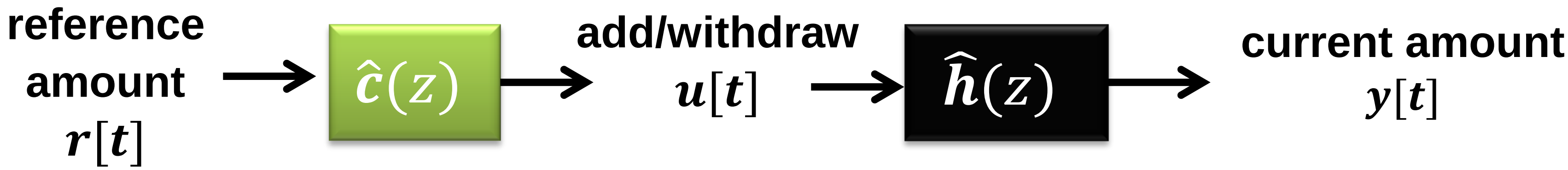
Bank deposit



$$\hat{h}(z) = \frac{1}{1 - 1.01z^{-1}}$$

$$\hat{c}(z) = \frac{1}{\hat{h}(z)}$$

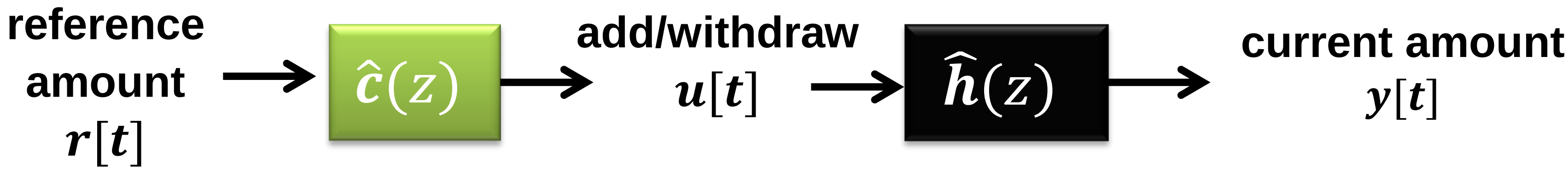
Bank deposit



$$\hat{h}(z) = \frac{1}{1 - 1.01z^{-1}}$$

$$\hat{c}(z) = \frac{1}{\hat{h}(z)} = 1 - 1.01z^{-1}$$

Bank deposit

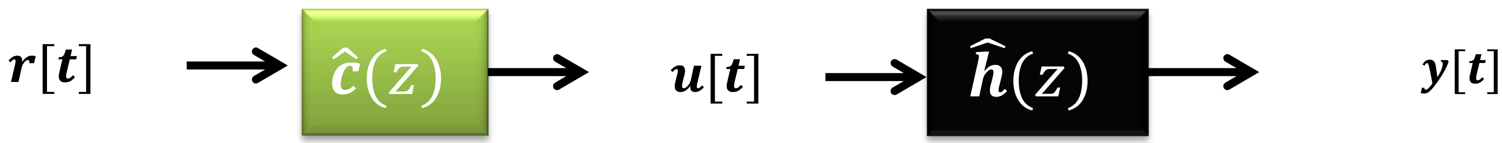


$$\hat{h}(z) = \frac{1}{1 - 1.01z^{-1}}$$

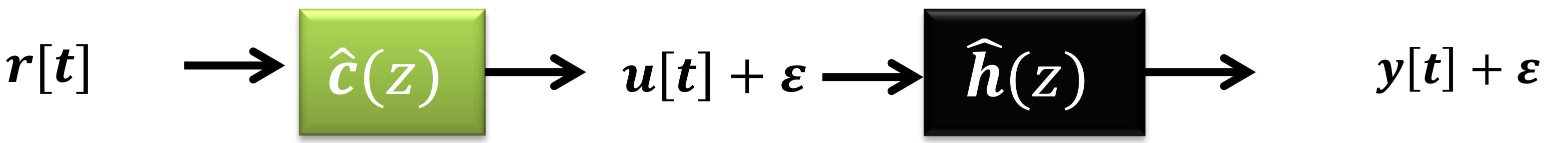
$$\hat{c}(z) = \frac{1}{\hat{h}(z)} = 1 - 1.01z^{-1}$$

$$u[t] := r[t] - 1.01r[t-1]$$

Open-loop control

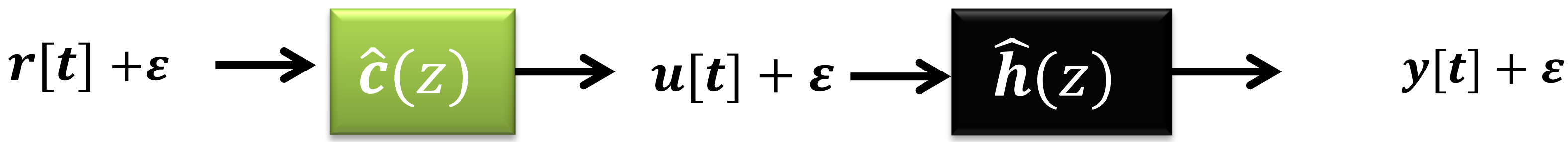


Open-loop control



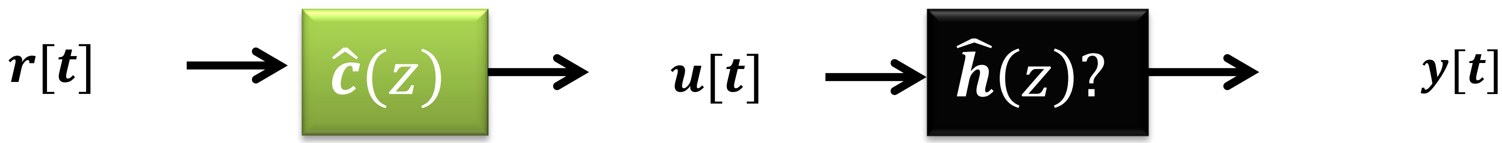
- Only makes sense for **BIBO-stable systems**

Open-loop control



- Only makes sense for **BIBO-stable systems**
- … and if the controller is also BIBO-stable

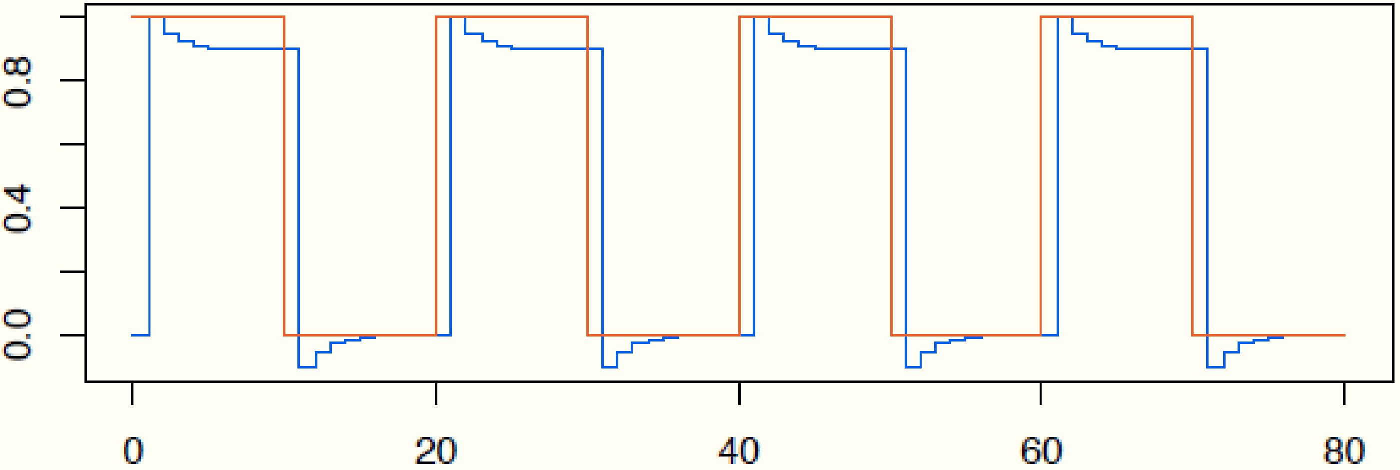
Open-loop control



- Only makes sense for **BIBO-stable systems**
- **Errors in system identification** will prevent exact tracking.

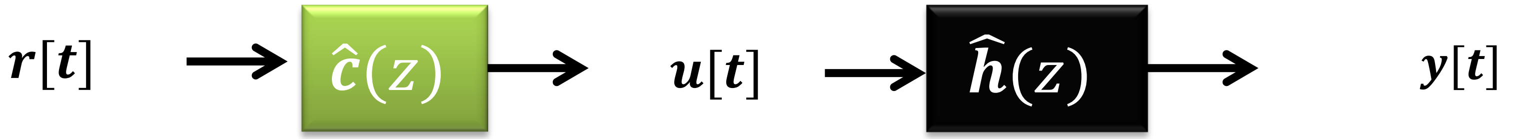
$$\widehat{\boldsymbol{h}}(z) = \frac{z^{-1}}{1 - 0.50z^{-1}}$$

$$\widetilde{\boldsymbol{h}}(z) = \frac{z^{-1}}{1 - 0.55z^{-1}}$$



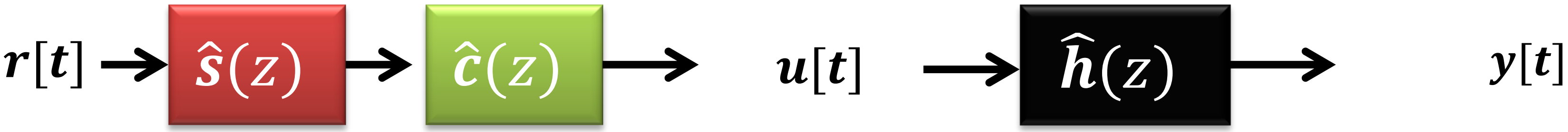
- **Errors in system identification** will prevent exact tracking.

Open-loop control



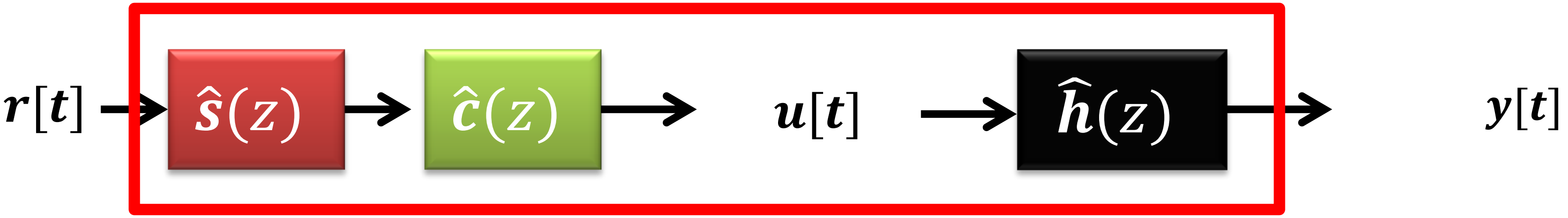
- Only makes sense for **BIBO-stable systems**
- **Errors in system identification** will prevent exact tracking.
- May require **smoothing of the reference signal.**

Open-loop control



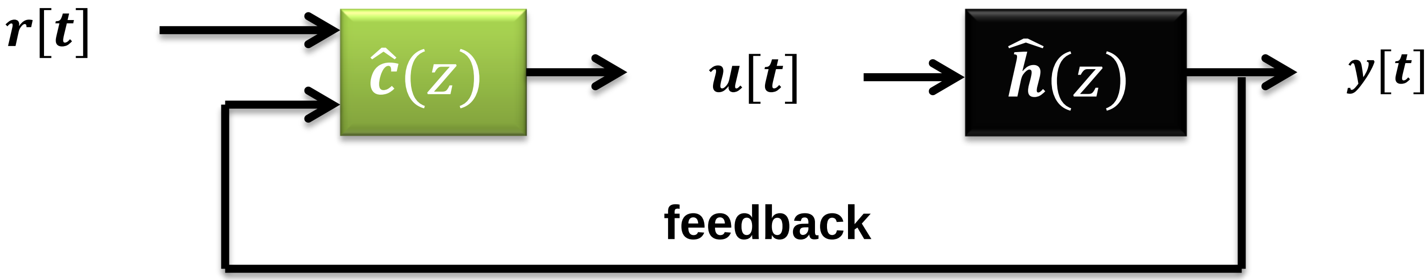
- Only makes sense for **BIBO-stable systems**
- **Errors in system identification** will prevent exact tracking.
- May require **smoothing of the reference signal.**

Open-loop control

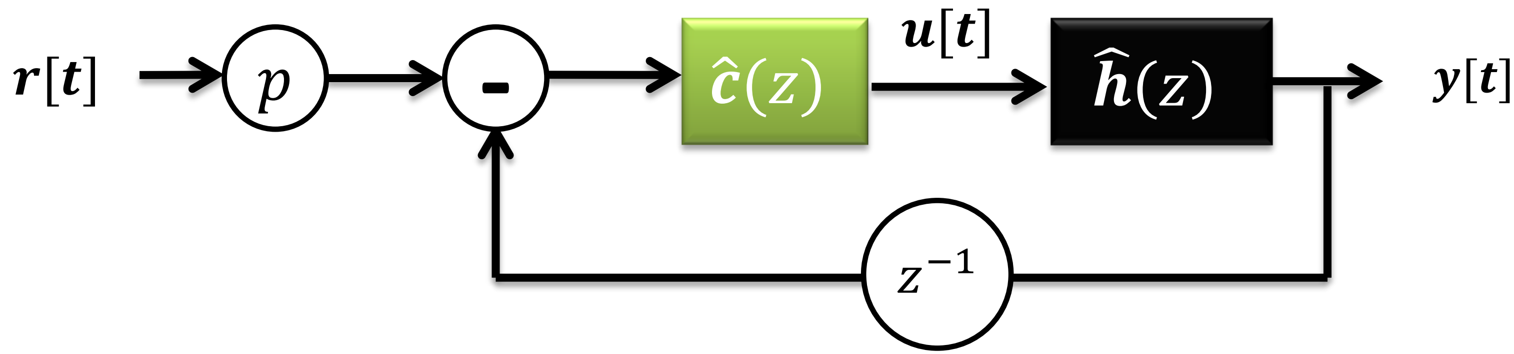


- Only makes sense for **BIBO-stable systems**
- **Errors in system identification** will prevent exact tracking.
- May require **smoothing of the reference signal.**
- Need to **analyze the combined system.**

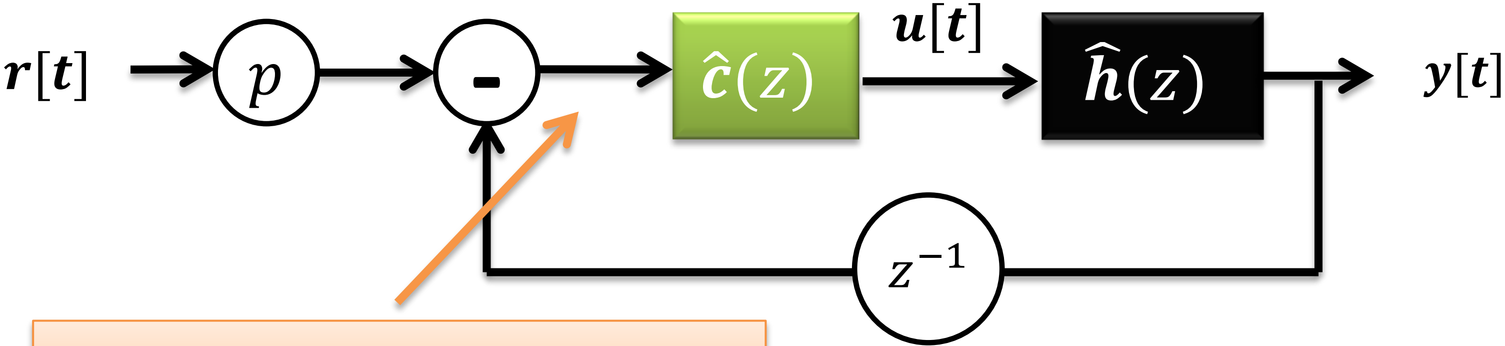
Closed-loop control



Unity feedback

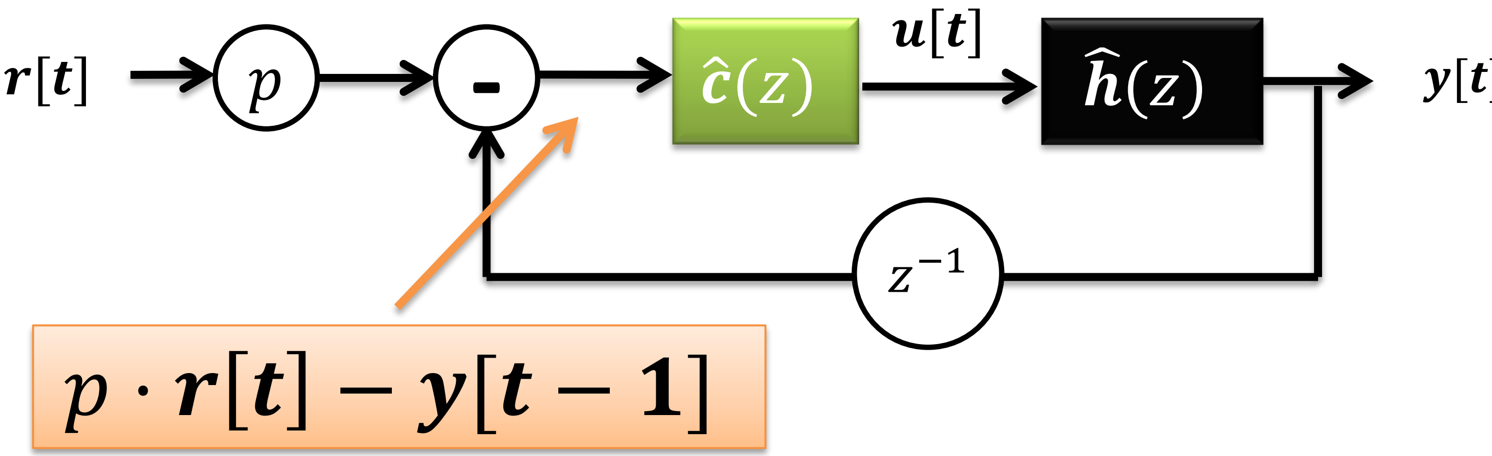


Unity feedback



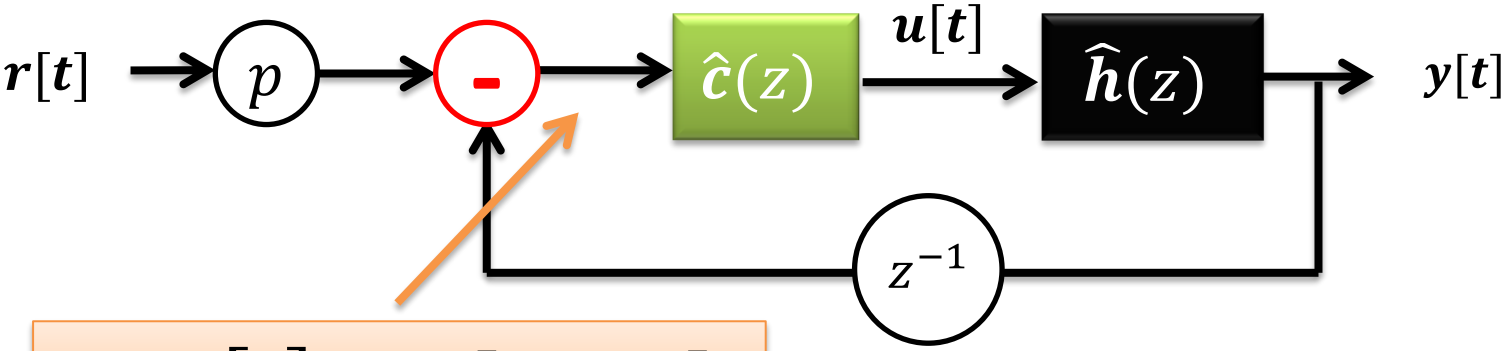
$$p \cdot \boldsymbol{r}[\boldsymbol{t}] - \boldsymbol{y}[\boldsymbol{t} - \mathbf{1}]$$

Unity feedback



$$\hat{y}(z) = \hat{h}(z)\hat{c}(z)(\ldots)$$

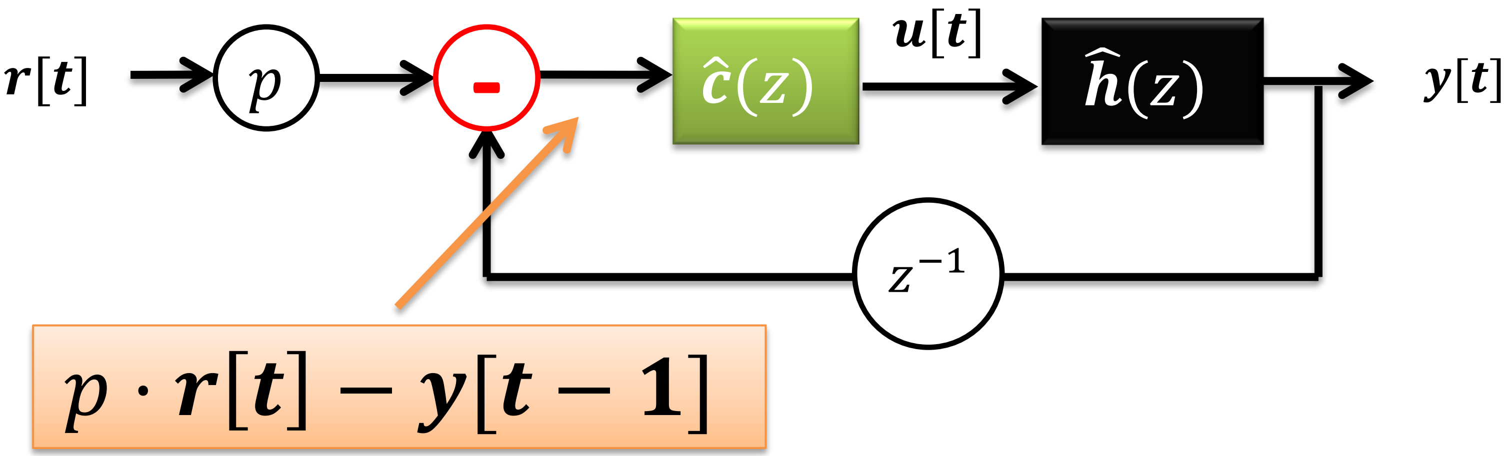
Unity feedback



$$p \cdot \boldsymbol{r}[\boldsymbol{t}] - \boldsymbol{y}[\boldsymbol{t} - \mathbf{1}]$$

$$\hat{\boldsymbol{y}}(z) = \hat{\boldsymbol{h}}(z)\hat{\boldsymbol{c}}(z)\left(p\hat{\boldsymbol{r}}(z) - z^{-1}\hat{\boldsymbol{y}}(z)\right)$$

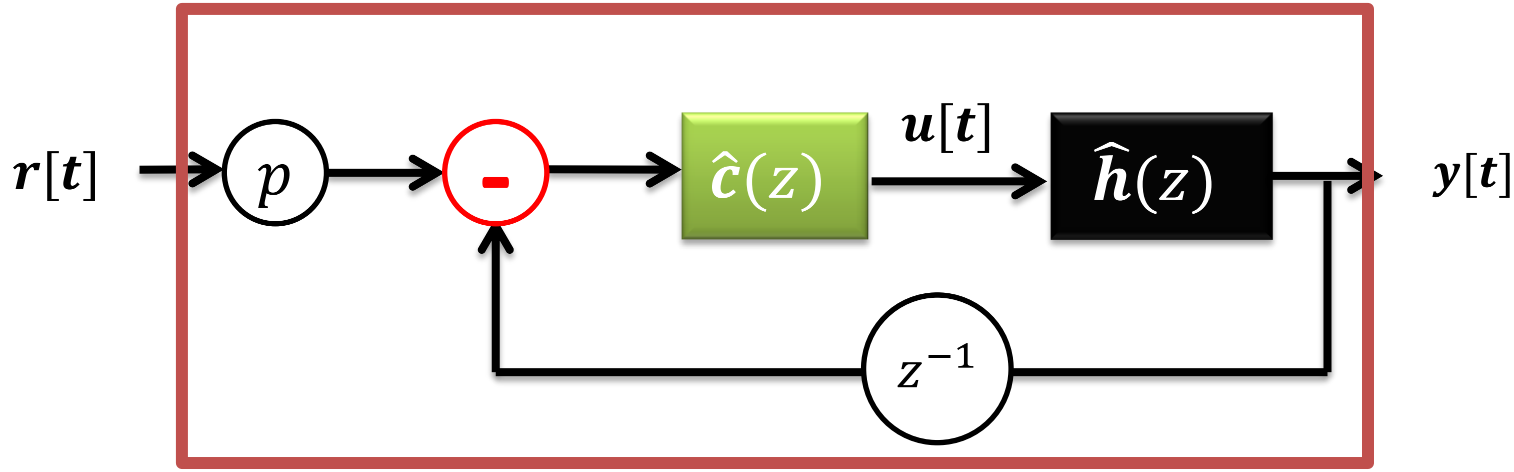
Unity feedback



$$\hat{\boldsymbol{y}}(z) = \hat{\boldsymbol{h}}(z)\hat{\boldsymbol{c}}(z)\left(p\hat{\boldsymbol{r}}(z) - z^{-1}\hat{\boldsymbol{y}}(z)\right)$$

$$\hat{\boldsymbol{y}}(z) = \frac{p\hat{\boldsymbol{h}}(z)\hat{\boldsymbol{c}}(z)}{1 + z^{-1}\hat{\boldsymbol{h}}(z)\hat{\boldsymbol{c}}(z)}\hat{\boldsymbol{r}}(z)$$

Unity feedback



$$\hat{\boldsymbol{y}}(z) = \hat{\boldsymbol{h}}(z)\hat{\boldsymbol{c}}(z)\left(p\hat{\boldsymbol{r}}(z) - z^{-1}\hat{\boldsymbol{y}}(z)\right)$$

$$\hat{\boldsymbol{g}}(z) = \frac{p\hat{\boldsymbol{h}}(z)\hat{\boldsymbol{c}}(z)}{1 + z^{-1}\hat{\boldsymbol{h}}(z)\hat{\boldsymbol{c}}(z)}$$

Unity feedback

$$\hat{\boldsymbol{g}}(z) = \frac{p\hat{\boldsymbol{h}}(z)\hat{\boldsymbol{c}}(z)}{1 + z^{-1}\hat{\boldsymbol{h}}(z)\hat{\boldsymbol{c}}(z)}$$

Unity feedback

$$\hat{\boldsymbol{g}}(z) = \frac{p\hat{\boldsymbol{h}}(z)\hat{\boldsymbol{c}}(z)}{1 + z^{-1}\hat{\boldsymbol{h}}(z)\hat{\boldsymbol{c}}(z)}$$

$$\hat{\boldsymbol{c}}(z) = -0.2\hat{\boldsymbol{h}}(z)^{-1}$$

$$p = -4$$

Unity feedback

$$\hat{\boldsymbol{g}}(z) = \frac{0.8}{1 - 0.2z^{-1}}$$

$$\hat{\boldsymbol{c}}(z) = -0.2\hat{\boldsymbol{h}}(z)^{-1}$$

$$p = -4$$

Unity feedback

$$\hat{\boldsymbol{g}}(z) = \frac{0.8}{1 - 0.2z^{-1}}$$

$$\hat{\boldsymbol{c}}(z) = -0.2\hat{\boldsymbol{h}}(z)^{-1} = -0.2(1 - 1.01z^{-1})$$

$$p = -4$$

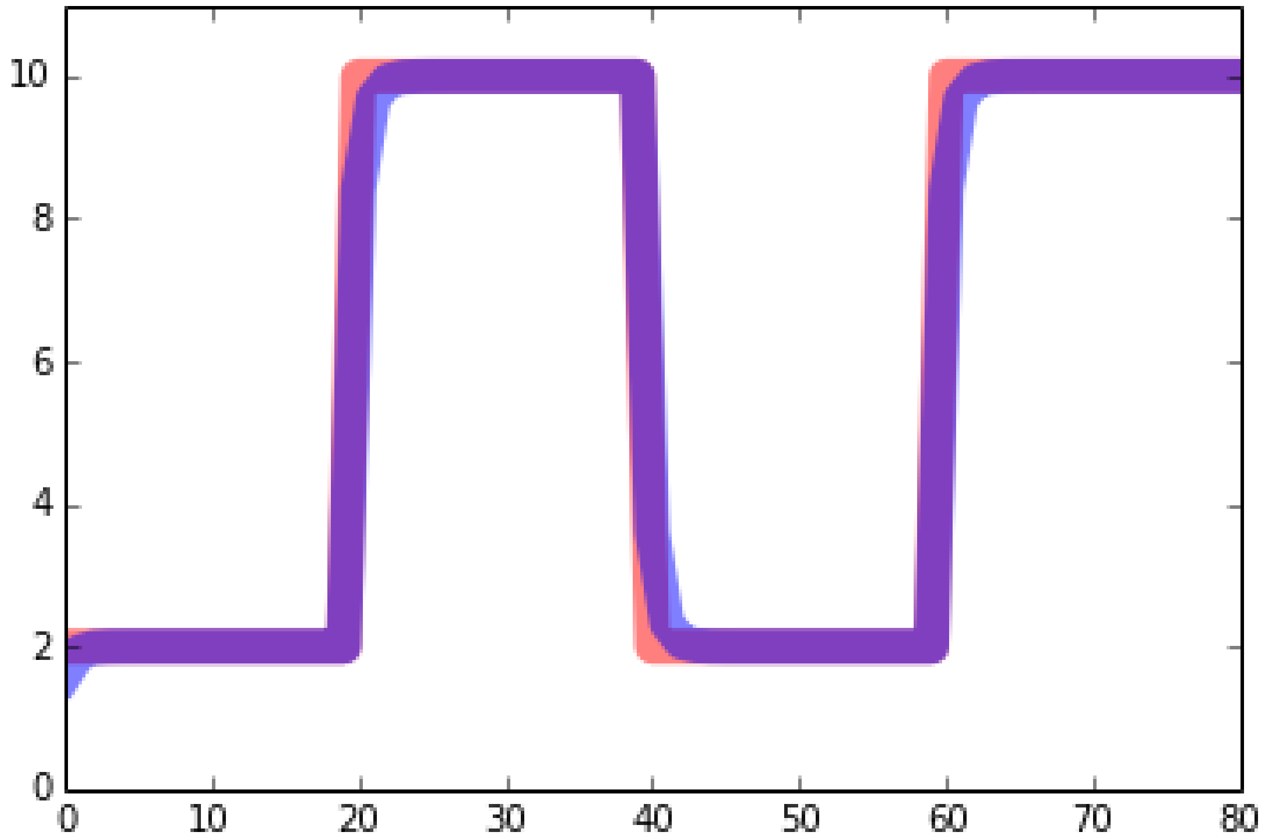
Unity feedback

0.8

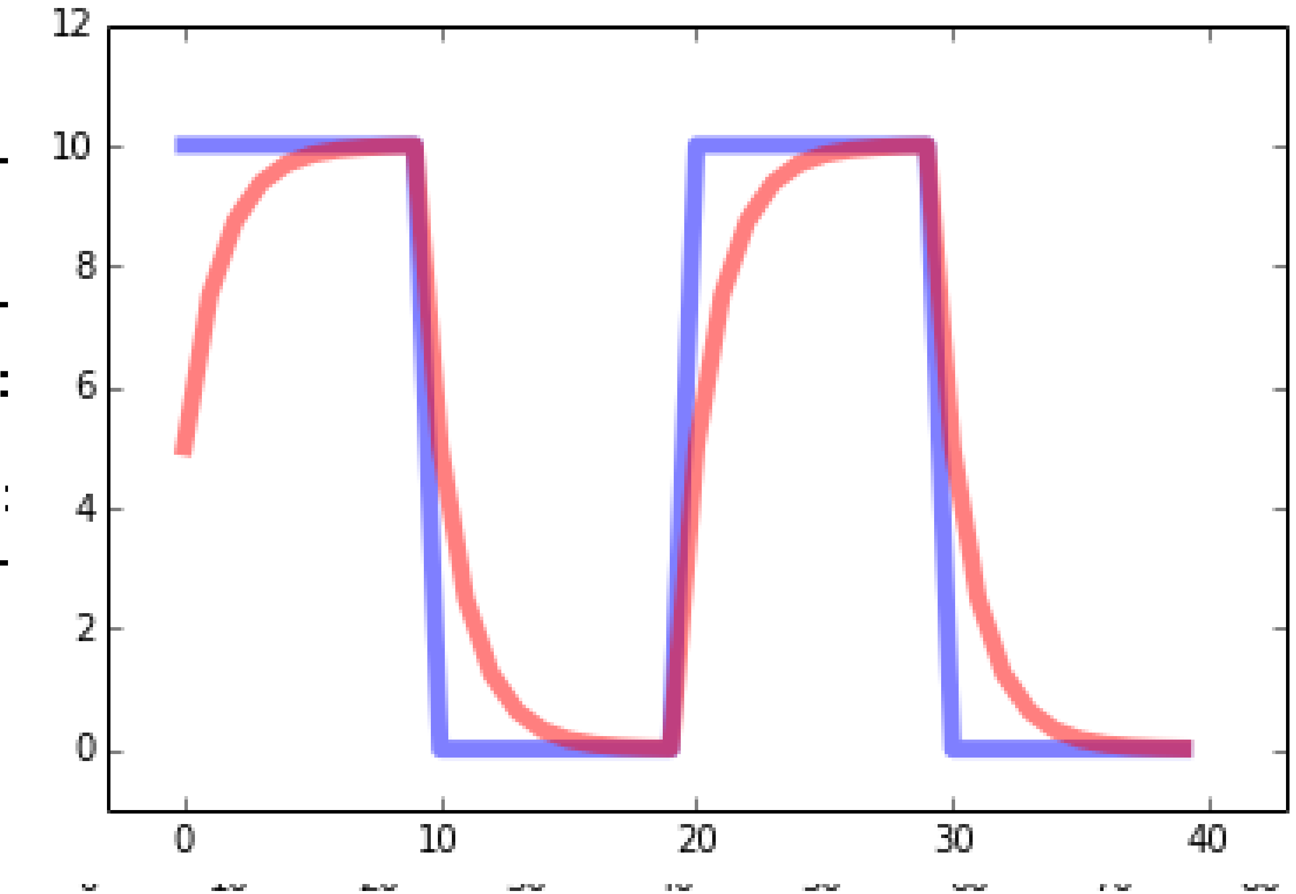
$\hat{c}(z)$:

$z^{-1})$

p :



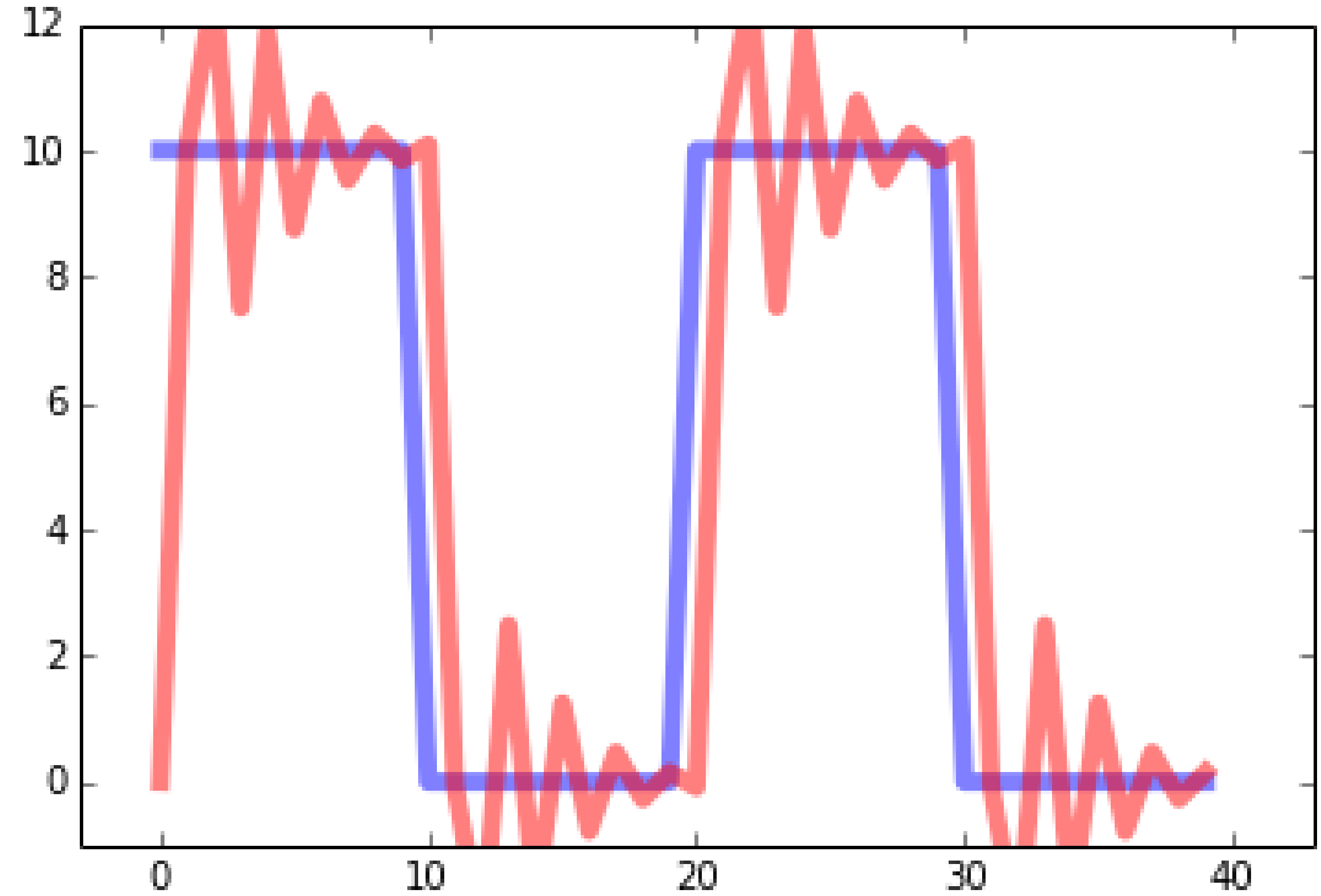
Unity feedback



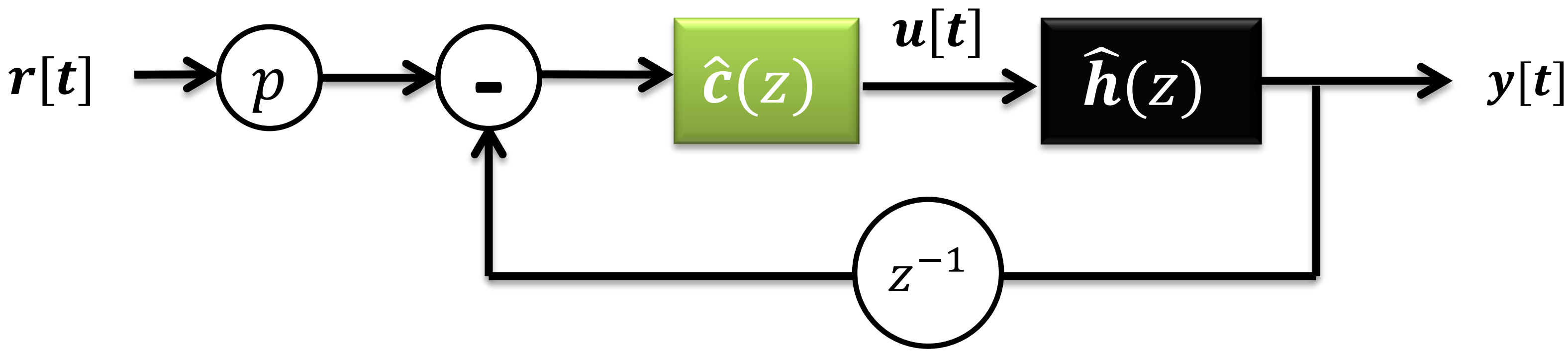
Unity feedback

$\hat{\boldsymbol{c}}(z)$ … $z^{-1})$

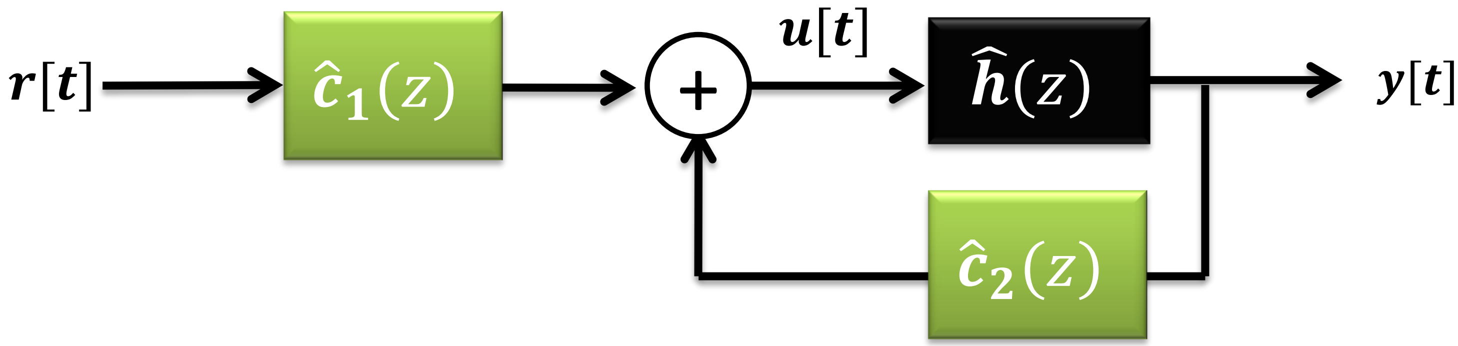
p



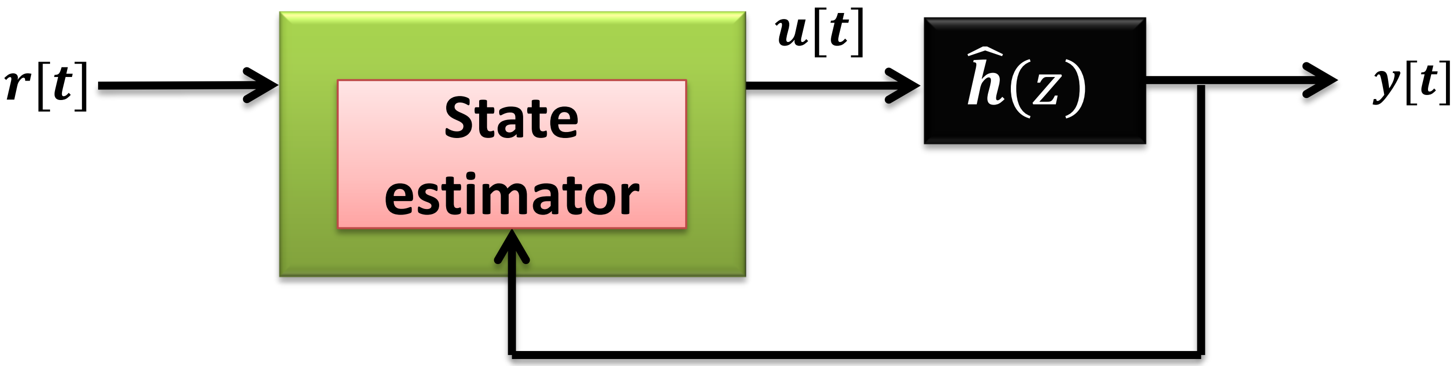
Unity feedback



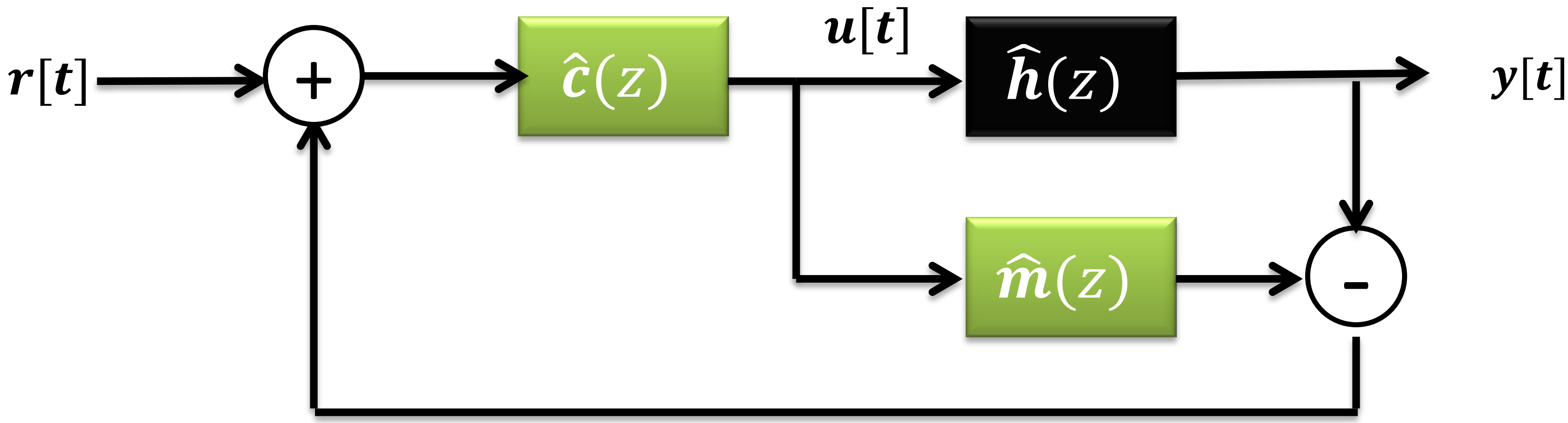
Model matching



State estimation



Internal model control



… and more

- Nonlinear control
- Feedback linearization
- Optimal control
- Predictive control
- Stochastic control
- Reinforcement learning
- …

$$x[t+1] = \boldsymbol{A}x[t] + \boldsymbol{B}u[t]$$
$$y[t] = \boldsymbol{C}x[t] + \boldsymbol{D}u[t]$$

$$\hat{\boldsymbol{y}}(z) = \hat{\boldsymbol{h}}(z)\hat{\boldsymbol{u}}(z)$$

$$\boldsymbol{y} = \boldsymbol{h} * \boldsymbol{u}$$

$$y[i] = \alpha_0 u[i] + \alpha_1 u[i-1] + \cdots$$

Discrete

Deterministic

Causal

Time-invariant

Finite-dimensional

Linear

Dynamic system

Convolution

Impulse response

Open-loop

Z-transform

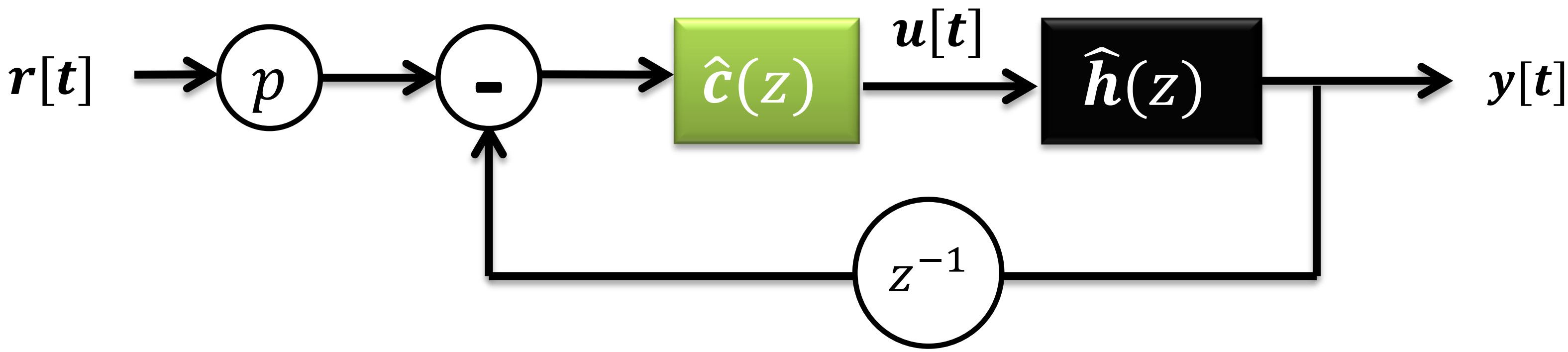
Transfer function

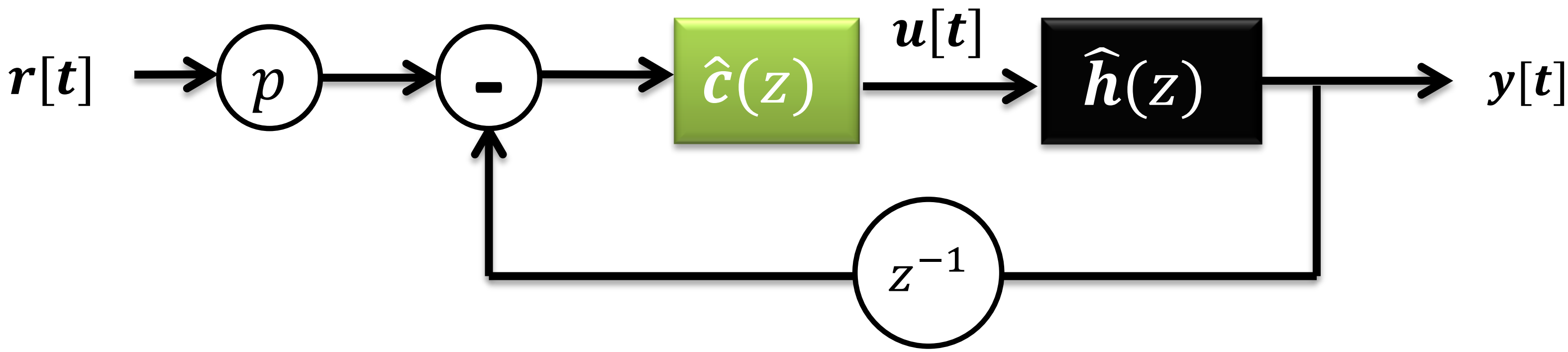
Closed-loop

FIR/IIR

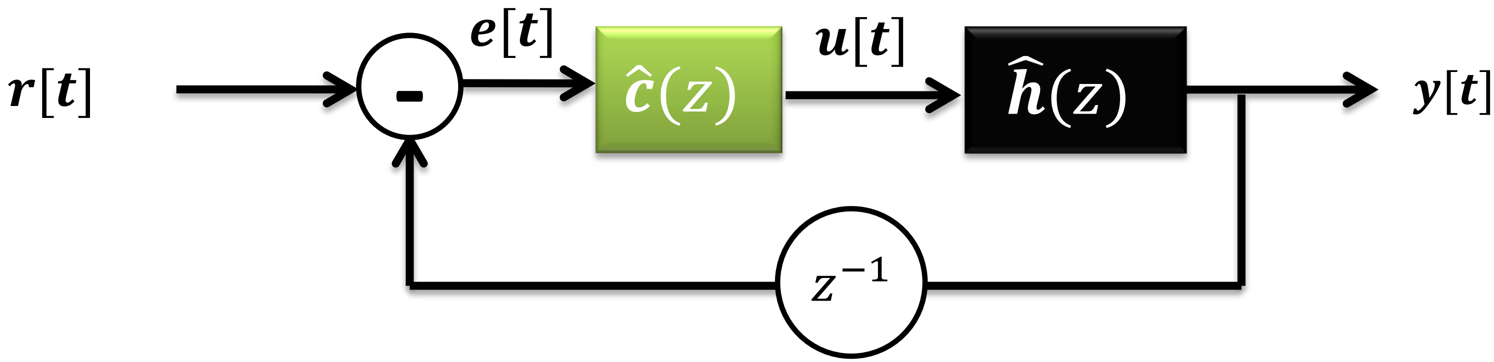
BIBO

Unity feedback

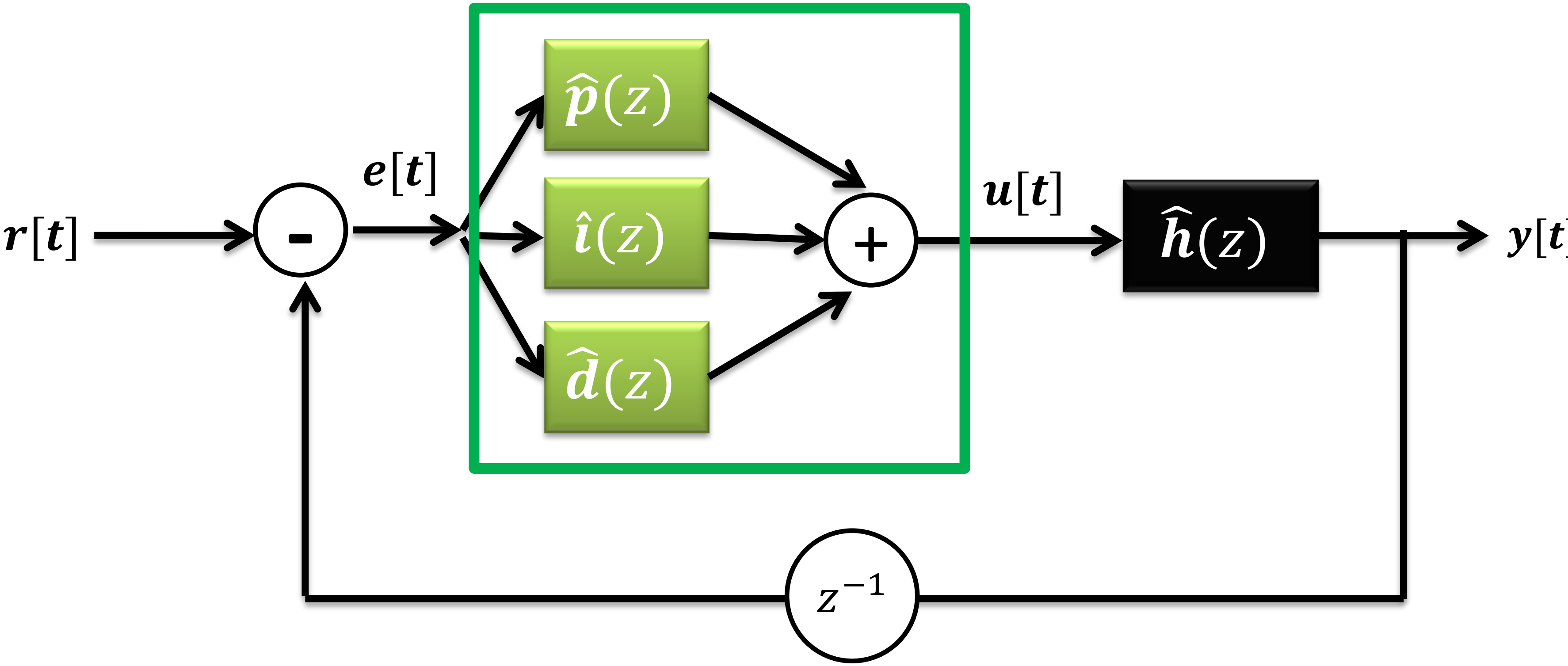




r[t]
p
-
ĉ(z)
u[t]
ĥ(z)
y[t]
z^−1

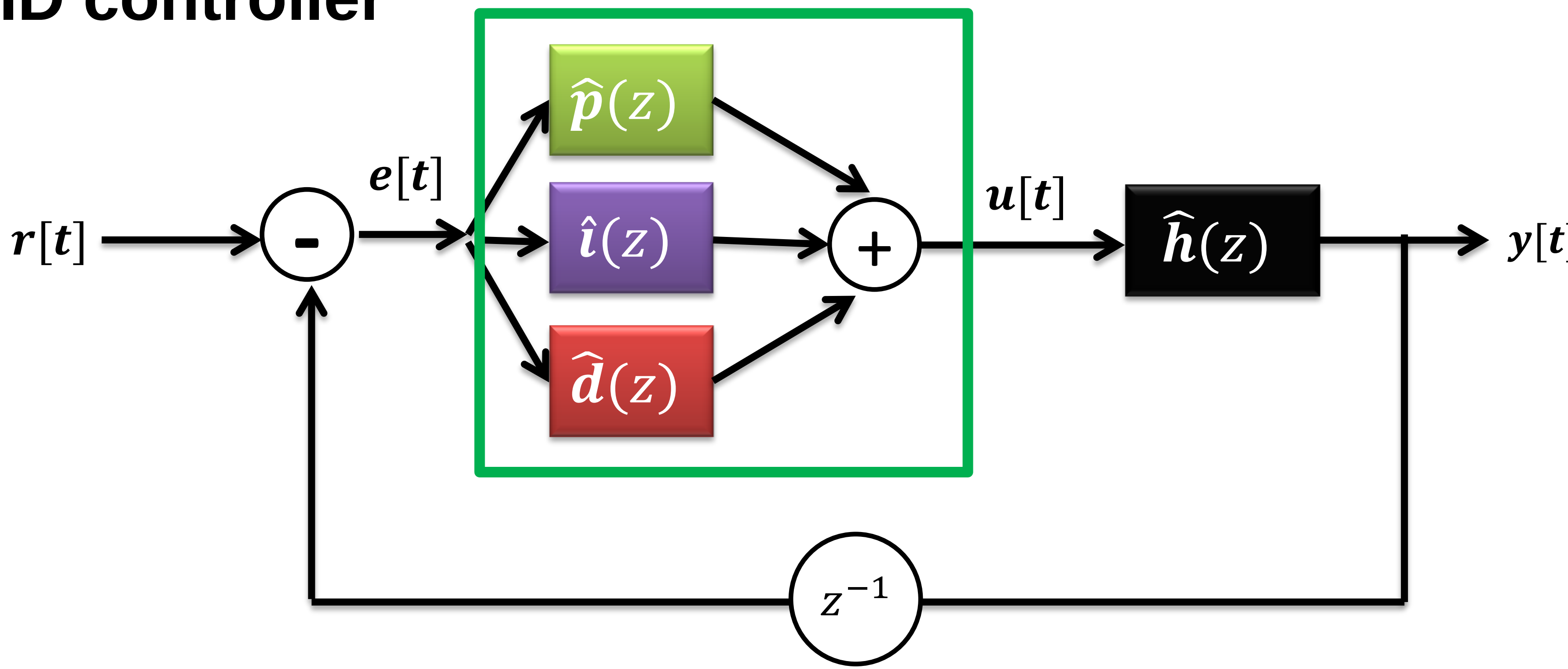


r[t]
-
e[t]
ĉ(z)
u[t]
ĥ(z)
y[t]
z^{−1}



r[t]
-
e[t]
p̂(z)
î(z)
d̂(z)
+
u[t]
ĥ(z)
y[t]
z^−1

PID controller



$$u[t] = K_p e[t] + K_i \sum_k e[k] + K_d(e[t] - e[t-1])$$

$$x[t+1] = \boldsymbol{A}x[t] + \boldsymbol{B}u[t]$$
$$y[t] = \boldsymbol{C}x[t] + \boldsymbol{D}u[t]$$

$$\widehat{\boldsymbol{y}}(z) = \widehat{\boldsymbol{h}}(z)\widehat{\boldsymbol{u}}(z)$$

$$y[i] = \alpha_0 u[i] + \alpha_1 u[i-1] + \cdots$$

$$\boldsymbol{y} = \boldsymbol{h} * \boldsymbol{u}$$

Discrete

Deterministic

Causal

Time-invariant

Finite-dimensional

Linear

Dynamic system

Convolution

Impulse response

Open-loop

Z-transform

Transfer function

Closed-loop

FIR/IIR

BIBO

PID controller